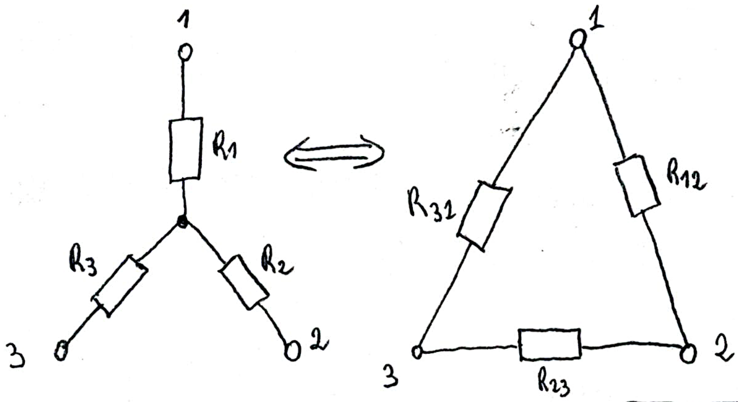
ELECTRICAL CIRCUITS 1 - CLASS NO. 02 (14.10.2024)

WYE-DELTA TRANSFORMATION



$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \qquad R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

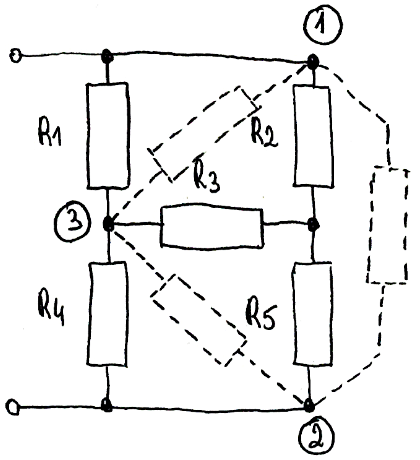
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}} \qquad R_{23} = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \qquad R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

if $R_1 = R_2 = R_3 = R$ then $\boxed{R_\Delta = 3R_\lambda}$ $\boxed{R_\lambda = \frac{1}{3} R_\Delta}$

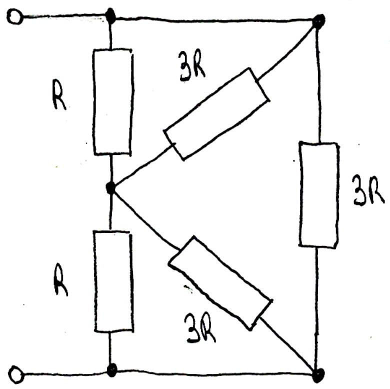
PROBLEM #1

Calculate the equivalent resistance of the circuit. $R_1 = R_2 = R_3 = R_4 = R_5 = R$



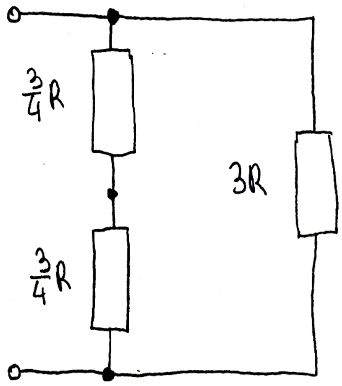
$R_\Delta = 3R_\lambda$

$\Rightarrow$



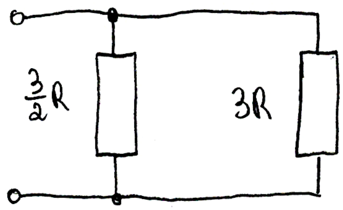
$\Rightarrow$

$$\frac{R \cdot 3R}{R + 3R} = \frac{3R^2}{4\cancel{R}} = \frac{3}{4} R$$



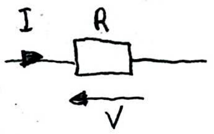
$$\frac{3}{4}R + \frac{3}{4}R = \frac{6}{4}R = \frac{3}{2}R$$

$\Rightarrow$



$$R_{EQ} = \frac{\frac{3}{2}R \cdot 3R}{\frac{3}{2}R + 3R} = \frac{\frac{9}{2}R^2}{\frac{9}{2}R} = \boxed{R}$$

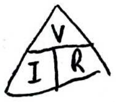
OHM'S LAW

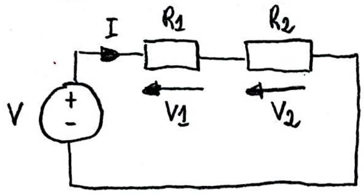


* for given resistance, the current is directly proportional to voltage

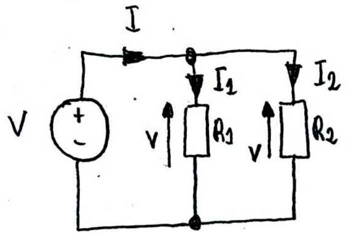
$I = \frac{V}{R}$ or $V = R \cdot I$ or $R = \frac{V}{I}$

* the simplest method to memorize Ohm's law is to use pyramid



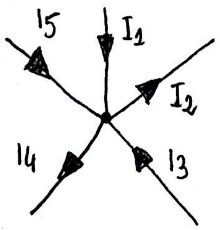


$I = \frac{V}{R_1 + R_2}$ $V_1 = I \cdot R_1$ $V_2 = I \cdot R_2$ $R_1 \neq R_2 \rightarrow V_1 \neq V_2$



$I_1 = \frac{V}{R_1}$ $I_2 = \frac{V}{R_2}$ $I = I_1 + I_2$

KCL - KIRCHHOFF'S CURRENT LAW



* the algebraic sum of the currents in a node is zero

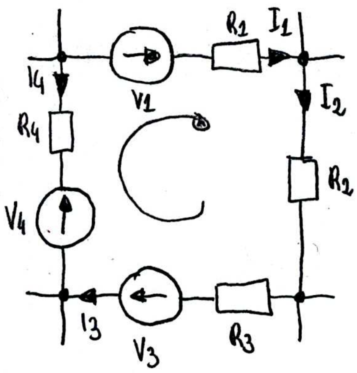
$\sum_K I_K = 0$ $I_1 - I_2 + I_3 - I_4 + I_5 = 0$

* if the current direction is into the node, we assign a positive sign to this current in an algebraic sum
* if the current direction is away from the node, we assign a negative sign to this current in an algebraic sum
* the sum of currents entering a node equals the sum of the currents leaving the node

$I_1 + I_3 + I_5 = I_2 + I_4$

* the algebraic sum of the currents entering a node is zero at every instant

KVL - KIRCHHOFF'S VOLTAGE LAW



* the algebraic sum of all the voltages around a loop is 0 (zero)

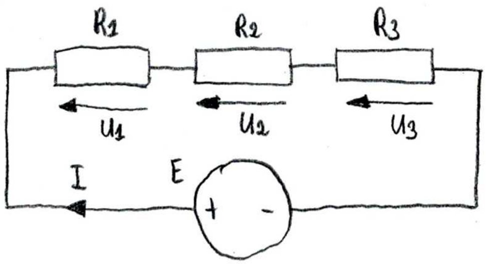
$V_1 - R_1 \cdot I_1 - R_2 \cdot I_2 - R_3 I_3 + V_3 + V_4 + I_4 \cdot R_4 = 0$

* in any complete electrical circuits, the algebraic sum of applied voltages must equal the algebraic sum of voltage drops

PROBLEM #2

Calculate the voltage drops and currents in the branches of the circuit shown in the figure. $E = 12V$, $R_1 = 1\,k\Omega$, $R_2 = 5\,k\Omega$, $R_3 = 10\,k\Omega$.

a)



$$I = \frac{E}{R_1 + R_2 + R_3} = \frac{12}{1000 + 5000 + 10\,000} = \frac{12}{16\,000} = 0.75\,mA$$

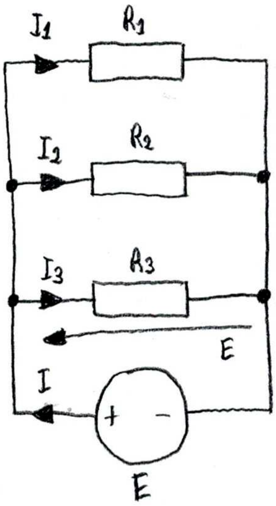
$$U_1 = R_1 \cdot I = 1000 \cdot 0.75\,mA = 0.75\,V$$

$$U_2 = R_2 \cdot I = 5000 \cdot 0.75\,mA = 3.75\,V$$

$$U_3 = R_3 \cdot I = 10000 \cdot 0.75\,mA = 7.5\,V$$

$$U_1 + U_2 + U_3 = 0.75 + 3.75 + 7.5 = 12V = E$$

b)



$$I_1 = \frac{E}{R_1} = \frac{12}{1000} = 12\,mA$$

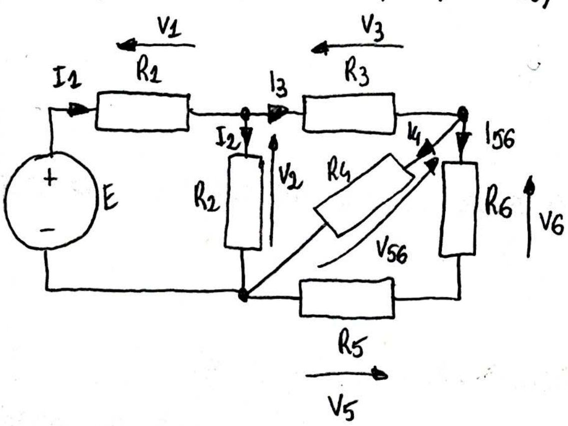
$$I_2 = \frac{E}{R_2} = \frac{12}{5000} = 2.4\,mA$$

$$I_3 = \frac{E}{R_3} = \frac{12}{10\,000} = 1.2\,mA$$

$$I = I_1 + I_2 + I_3 = 12\,mA + 2.4\,mA + 1.2\,mA = 15.6\,mA$$

PROBLEM #3

The voltage drop across R_6 is 30V. Calculate the voltage E, the current through a voltage source and the equivalent resistance of the circuit.
$R_1 = 2.4\Omega$, $R_2 = 4\Omega$, $R_3 = 1\Omega$, $R_4 = 2.5\Omega$, $R_5 = 2\Omega$, $R_6 = 3\Omega$.



$$V_6 = 30V$$

$$I_{56} = \frac{V_6}{R_6} = \frac{30}{3} = 10A \qquad V_5 = R_5 \cdot I_{56} = 2 \cdot 10 = 20V$$

$$V_{56} = V_5 + V_6 = 20 + 30 = 50\,V$$

$$I_4 = \frac{V_{56}}{R_4} = \frac{50}{2{,}5} = 20A \qquad I_3 = I_4 + I_{56} = 20 + 10 = 30A$$

$$V_3 = I_3 \cdot R_3 = 30 \cdot 1 = 30V \qquad V_2 = V_3 + V_{56} = 30 + 50 = 80V$$

$$I_2 = \frac{V_2}{R_2} = \frac{80}{4} = 20\,A \qquad I_1 = I_2 + I_3 = 20 + 30 = \boxed{50A}$$

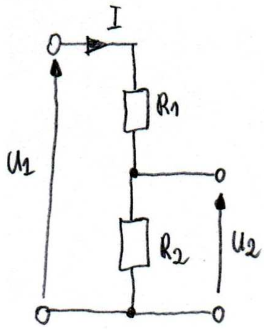
$$V_1 = R_1 \cdot I_1 = 2.4 \cdot 50 = 120V$$

$$E = V_1 + V_2 = 120 + 80 = \boxed{200V}$$

$$R_{EQ} = \frac{E}{I_1} = \frac{200}{5} = \boxed{4\Omega}$$

PROBLEM #4

Calculate the voltage U_2 for the circuit shown in figure. $U_1 = 100V$, $R_1 = 70\Omega$, $R_2 = 30\Omega$.



$$I = \frac{U_1}{R_1+R_2} = \frac{100}{70+30} = \frac{100}{100} = 1\ A$$

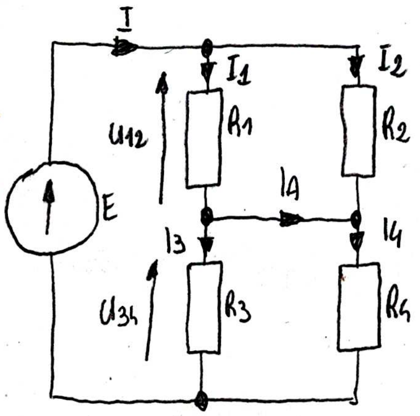
$$U_2 = R_2 \cdot I = 30 \cdot 1 = \boxed{30V}$$

$$U_2 = \frac{R_2}{R_1+R_2} U_1$$

$$U_2 = \frac{30}{70+30} \cdot 100 = \boxed{30\ V}$$

PROBLEM #5

Calculate the current I_A in the circuit presented in the figure.
$E = 120V$, $R_1 = 10\Omega$, $R_2 = 15\Omega$, $R_3 = 60\Omega$, $R_4 = 40\Omega$

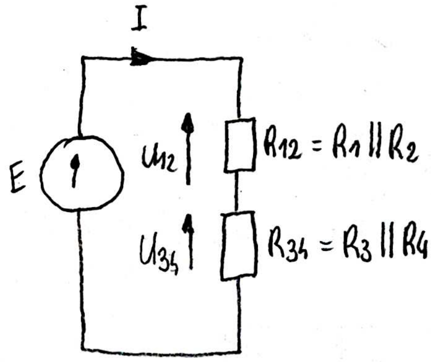


$R_1 \parallel R_2$

$R_3 \parallel R_4$

$I_A = I_1 - I_3$

$\Rightarrow$



$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{10 \cdot 15}{10+15} = \frac{150}{25} = 6\Omega$$

$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4} = \frac{60 \cdot 40}{60+40} = \frac{2400}{100} = 24\Omega$$

$$I = \frac{E}{R_{12}+R_{34}} = \frac{120}{6+24} = \frac{120}{30} = 4A$$

$$U_{12} = R_{12} \cdot I = 6 \cdot 4 = 24V$$

$$U_{34} = R_{34} \cdot I = 24 \cdot 4 = 96V$$

$$I_1 = \frac{U_{12}}{R_1} = \frac{24}{10} = 2.4A$$

$$I_3 = \frac{U_{34}}{R_3} = \frac{96}{60} = 1.6A$$

$$I_A = I_1 - I_3 = 2.4 - 1.6 = \boxed{0.8\ A}$$