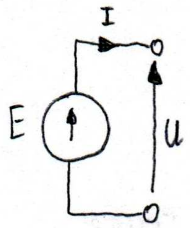
ELECTRICAL CIRCUITS 1 - CLASS NO. 03 (21.10.2024)

VOLTAGE SOURCE

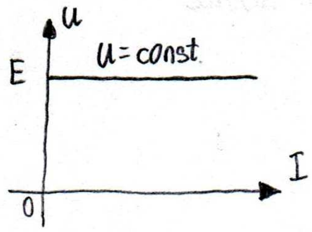
the ideal voltage source



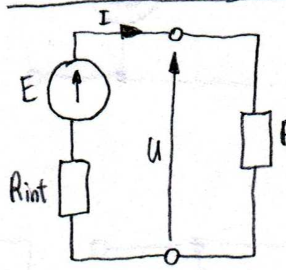
* an internal resistance is eqaul to 0

  $R_{int} = 0$

* constant voltage



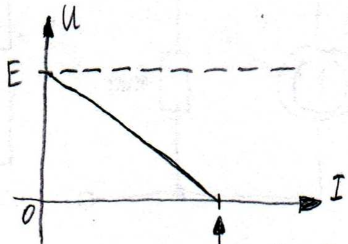
the real voltage source



U - terminal voltage

R_L - load

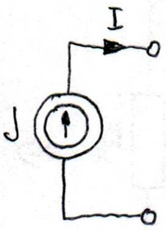
$$\boxed{U = E - R_{int} \cdot I}$$



$R_L = 0, \quad I = \frac{E}{R_{int}}$

CURRENT SOURCE

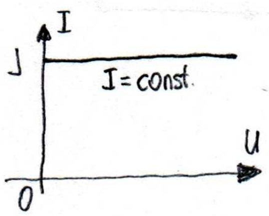
the ideal current source



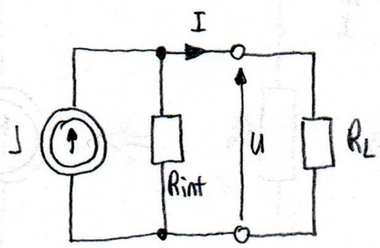
* an internal resistance is equal to infinity

  $R_{int} = \infty, \quad G_{int} = 0$

* constant current

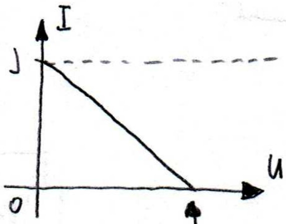


the real current source



R_L - load

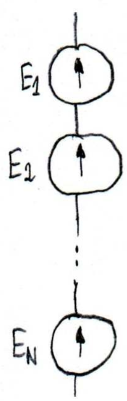
$$\boxed{I = J - \frac{U}{R_{int}}}$$



$R_L = \infty, \quad U = R_{int} \cdot J$

VOLTAGE SOURCES CONNECTED IN SERIES

* the total effective source voltage is the sum of the individual source voltages

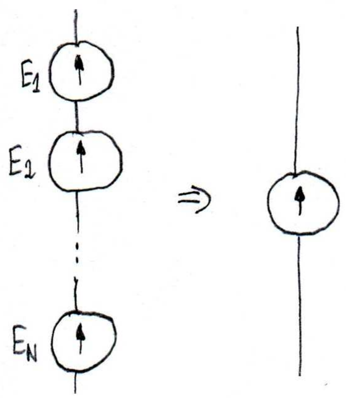


$\Rightarrow$

$E_X = E_1 + E_2 + \ldots + E_N$

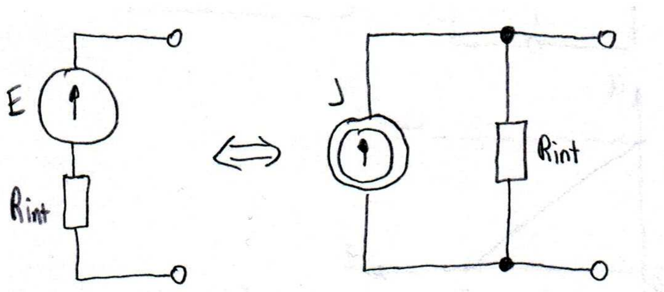
CURRENT SOURCES CONNECTED IN PARALLEL

* the total effective source voltage is the sum of the individual source voltages



$E_X = E_1 + E_2 + \ldots + E_N$

SOURCE CONVERSION



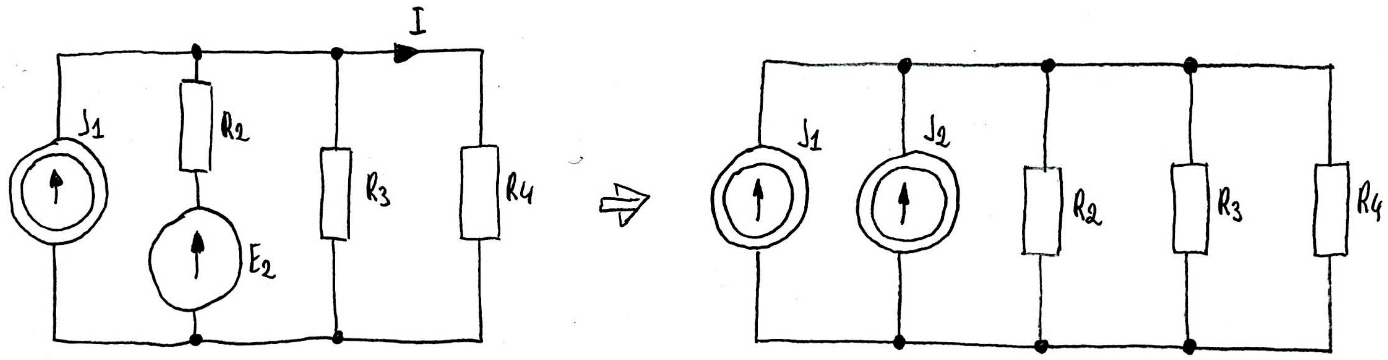
$\boxed{J = \frac{E}{R_{int}}}$

$\boxed{E = J \cdot R_{int}}$

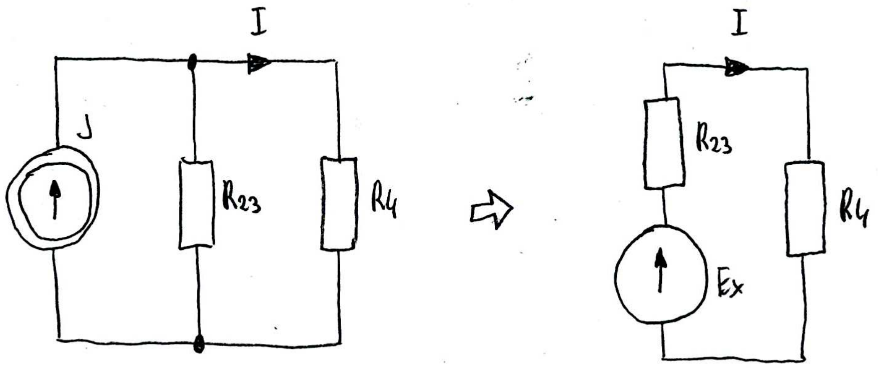
* to replace a voltage source with current source

PROBLEM #1

What should be the minimum power rating of the resistor R_4 in the circuit shown in the figure?. $J_1 = 1A$, $E_2 = 10V$, $R_2 = R_3 = R_4 = 2\Omega$



$J_2 = \frac{E_2}{R_2} = \frac{10}{2} = 5A$



$E_x = J \cdot R_{23} = 6 \cdot 1 = 6V$

$I = \frac{E_x}{R_{23} + R_4} = \frac{6}{1+2} = \frac{6}{3} = 2A$

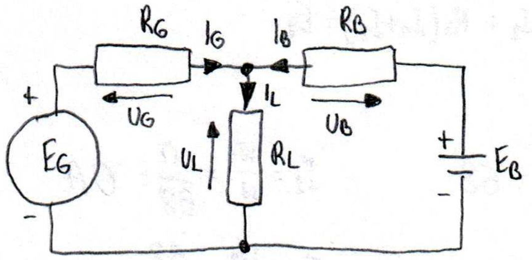
$P_{min} = R_4 \cdot I^2 = 2 \cdot 2^2 = \boxed{8\ W}$

$J = J_1 + J_2 = 1 + 5 = 6A$

$R_{23} = R_2 \| R_3 = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{2 \cdot 2}{2+2} = 1\,\Omega$

PROBLEM #2

An automobile generator with an internal resistance of $R_G = 0.2\Omega$ develops an open-circuit voltage of $E_G = 16V$. The storage battery has an internal resistance of $R_B = 0.1\Omega$ and an open-circuit voltage of $E_B = 12.8V$. Both sources are connected in parallel to a $R_L = 1\Omega$ load. Determine the load current.



KCL : $\begin{cases} I_G + I_B = I_L \end{cases}$

KVL : $\begin{cases} U_G + U_L = E_G \\ U_B + U_L = E_B \end{cases} \Rightarrow \begin{cases} R_G \cdot I_G + R_L \cdot I_L = E_G \\ R_B \cdot I_B + R_L \cdot I_L = E_B \end{cases}$

$$\begin{cases} R_G \cdot I_G + R_L(I_G + I_B) = E_G \\ R_B \cdot I_B + R_L(I_G + I_B) = E_B \end{cases} \Rightarrow \begin{cases} (R_L + R_G) I_G + R_L \cdot I_B = E_G \\ R_L \cdot I_G + (R_B + R_L) \cdot I_B = E_B \end{cases}$$

$$\begin{cases} (1+0.2) I_G + 1 \cdot I_B = 16 \\ 1 \cdot I_G + (1+0.1) = 12.8 \end{cases} \Rightarrow \begin{cases} 1.2 I_G + I_B = 16 \\ I_G + 1.1 I_B = 12.8 \end{cases}$$

$$W = \begin{vmatrix} 1.2 & 1 \\ 1 & 1.1 \end{vmatrix} = 1.2 \cdot 1.1 - 1 \cdot 1 = 1.32 - 1 = 0.32$$

$$W_G = \begin{vmatrix} 16 & 1 \\ 12.8 & 1.1 \end{vmatrix} = 16 \cdot 1.1 - 12.8 \cdot 1 = 17.6 - 12.8 = 4.8$$

$$W_B = \begin{vmatrix} 1.2 & 16 \\ 1 & 12.8 \end{vmatrix} = 1.2 \cdot 12.8 - 1 \cdot 16 = 15.36 - 16 = -0.64$$

$$I_G = \frac{W_G}{W} = \frac{4.8}{0.32} = 15 A$$

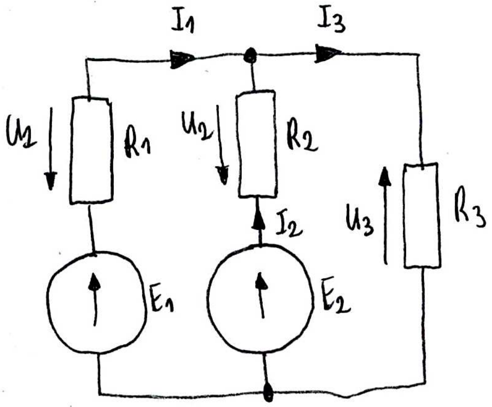
$$I_B = \frac{W_B}{W} = \frac{-0.64}{0.32} = -2A$$

$$I_L = I_G + I_B = 15 - 2 = \boxed{13 A}$$

PROBLEM #3

Make a power balance for the circuit shown in the figure (i.e., the power provided by the batteries equals the sum of the power dissipated by the resistances).

$E_1 = 10V$, $R_1 = 4\Omega$, $E_2 = 12V$, $R_2 = 2\Omega$, $R_3 = 10\Omega$



$$\begin{cases} I_1 + I_2 = I_3 \\ R_1 I_1 - R_2 I_2 = E_1 - E_2 \\ R_2 I_2 + R_3 I_3 = E_2 \end{cases} \Rightarrow \begin{cases} R_1 I_1 - R_2 I_2 = E_1 - E_2 \\ R_2 I_2 + R_3(I_1 + I_2) = E_2 \end{cases} \Rightarrow$$

$$\begin{cases} R_1 I_1 - R_2 I_2 = E_1 - E_2 \\ R_3 I_1 + (R_2 + R_3) I_2 = E_2 \end{cases} \quad \begin{cases} 4 I_1 - 2 I_2 = 10 - 12 \\ 10 I_1 + (2+10) I_2 = 12 \end{cases}$$

$$\begin{cases} 4I_1 - 2I_2 = -2 \\ 10I_1 + 12I_2 = 12 \end{cases}$$

$$W = \begin{vmatrix} 4 & -2 \\ 10 & 12 \end{vmatrix} = 4\cdot 12 - (-2)\cdot 10 = 48 + 20 = 68$$

$$W_1 = \begin{vmatrix} -2 & -2 \\ 12 & 12 \end{vmatrix} = -2\cdot 12 - (-2)\cdot 12 = -48 + 48 = 0$$

$$W_2 = \begin{vmatrix} 4 & -2 \\ 10 & 12 \end{vmatrix} = 4\cdot 12 - (-2)\cdot 10 = 48 + 20 = 68$$

$$I_1 = \frac{W_1}{W} = \frac{0}{68} = 0\,A$$

$$I_2 = \frac{W_2}{W} = \frac{68}{68} = 1\,A$$

$$I_3 = I_1 + I_2 = 1\,A$$

Power of batteries: $P_{E1} + P_{E2} = E_1 \cdot I_1 + E_2 \cdot I_2 = 10\cdot 0 + 12\cdot 1 = \boxed{12\,W}$

Power of resistances: $P_{R1} + P_{R2} + P_{R3} = R_1 I_1^2 + R_2 \cdot I_2^2 + R_3 \cdot I_3^2 = 4\cdot 0^2 + 2\cdot 1^2 + 10\cdot 1^2 = 0 + 2 + 10 = \boxed{12\,W}$