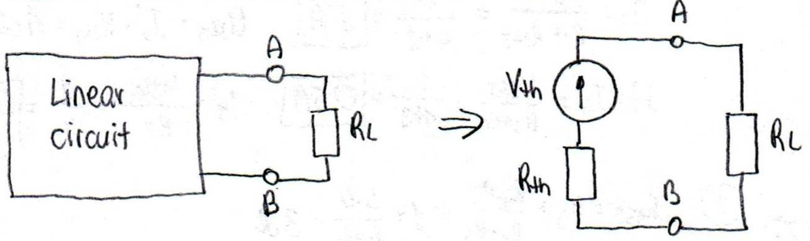
ELECTRICAL CIRCUITS 1 - CLASS NO. 08 (02.12.2024)

THEVENIN'S THEOREM

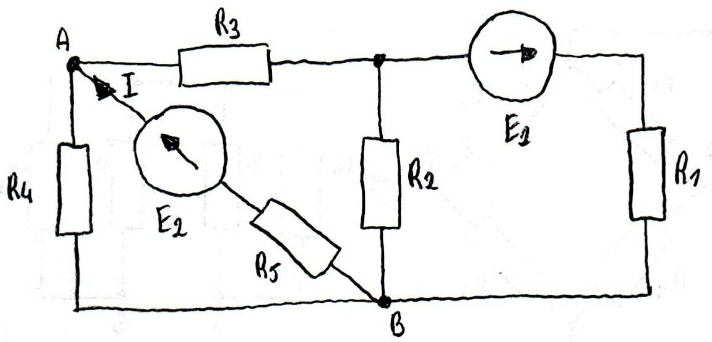
* any linear circuit containing only voltage sources, current sources, resistances can be replaced at terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}



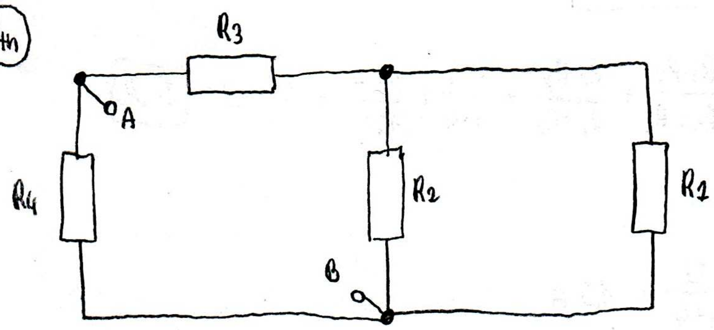
V_{th} - the voltage obtained at terminals A-B of the circuit with terminals A-B open circuited

R_{th} - the resistance between terminals A-B when all ideals voltage sources are replaced by a short circuit and all current sources are replaced by an open circuit

PROBLEM #1



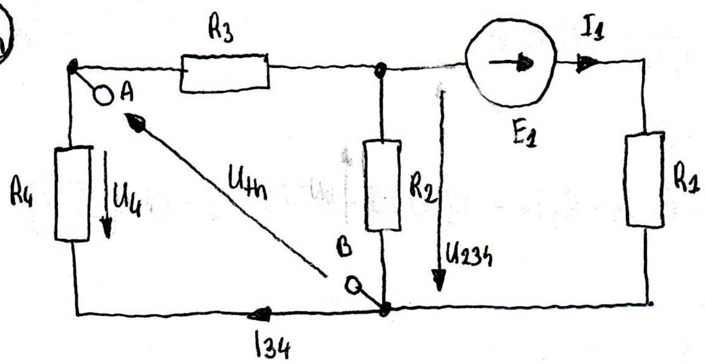
R_{th}



$$R_{123} = \frac{R_1 \cdot R_2}{R_1 + R_2} + R_3 = \frac{2 \cdot 3}{2+3} + 4 = 5,2\ \Omega$$

$$R_{th} = \frac{R_4 \cdot R_{123}}{R_4 + R_{123}} = \frac{2 \cdot 5,2}{2 + 5,2} = \boxed{1,44\ \Omega}$$

U_{th}



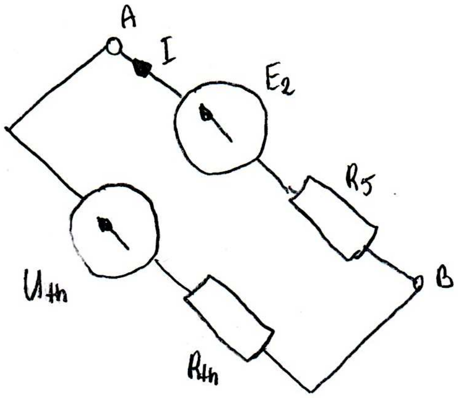
$$R_{234} = \frac{(R_3 + R_4) \cdot R_2}{R_3 + R_4 + R_2} = \frac{(4+2) \cdot 3}{4+2+3} = 2\ \Omega$$

$$I_1 = \frac{E_1}{R_{234} + R_1} = \frac{14}{2+2} = 3,5\,A$$

$$U_{234} = I_1 \cdot R_{234} = 3,5 \cdot 2 = 7\,V$$

$$I_{34} = \frac{U_{234}}{R_3 + R_4} = \frac{7}{4+2} = 1.167\,A$$

$$U_4 = I_{34} \cdot R_4 = 1.167 \cdot 2 = 2.33\,V$$



KVL: $U_{th} + U_4 = 0$

$U_{th} = -U_4 = \boxed{-2.33\,V}$

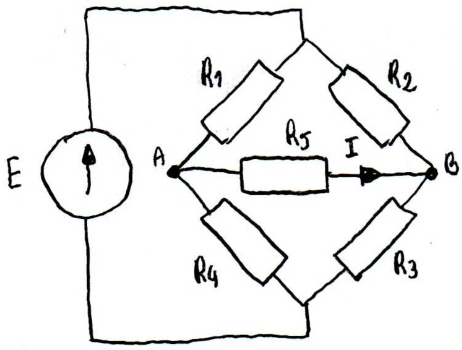
$$I = \frac{E_2 - U_{th}}{R_5 + R_{th}} = \frac{20-(-2.33)}{6+1.44} = \frac{22.33}{7.44} = \boxed{3A}$$

$$P_5 = R_5 \cdot I^2 = 6 \cdot 3^2 = 6 \cdot 9 = \boxed{54\,W}$$

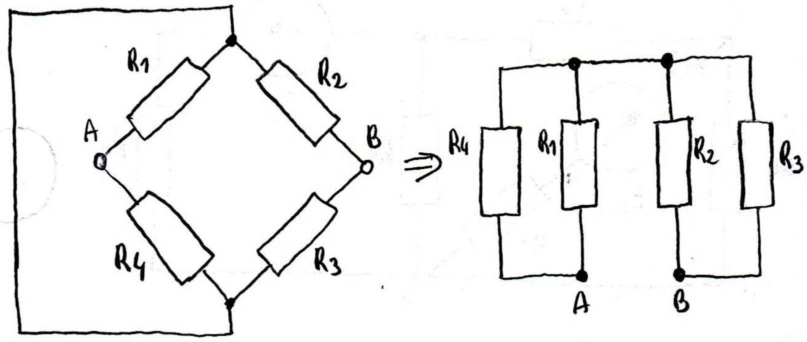
PROBLEM #2

What should be the minimum power rating of the resistor R_5 in the circuit shown in the figure? Use Thevenin's theorem.

$R_1 = 4\,\Omega$, $R_2 = 12\,\Omega$, $R_3 = 4\,\Omega$, $R_4 = 4\,\Omega$, $R_5 = 1\,\Omega$, $E = 12\,V$

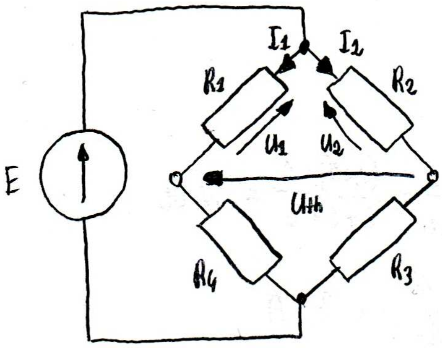


R_{th}



$$R_{th} = \frac{R_1 \cdot R_4}{R_1 + R_4} + \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{4 \cdot 4}{4+4} + \frac{12 \cdot 4}{12+4} = 2 + 3 = \boxed{5\,\Omega}$$

U_{th}

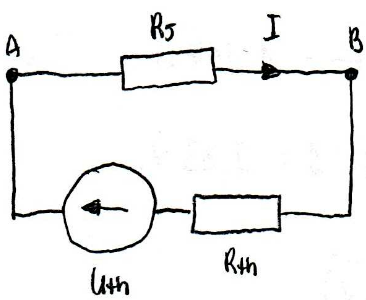


$$I_1 = \frac{E}{R_1 + R_4} = \frac{12}{4+4} = 1.5\,A$$

$$I_2 = \frac{E}{R_2 + R_3} = \frac{12}{12+4} = 0.75\,A$$

$U_{th} + U_1 - U_2 = 0$

$U_{th} = U_2 - U_1 = R_2 \cdot I_2 - R_1 I_1 = 12 \cdot 0.75 - 4 \cdot 1.5 = 9 - 6 = \boxed{3\,V}$

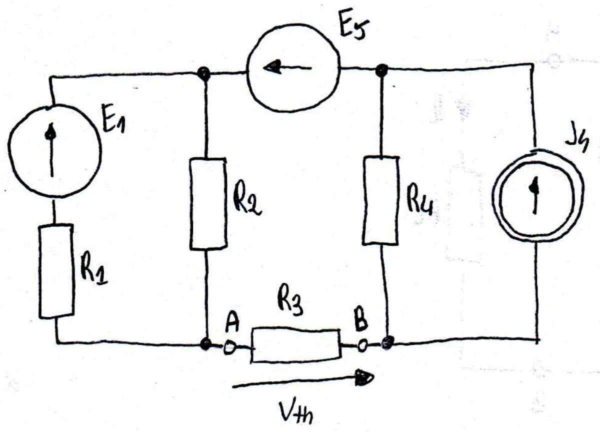


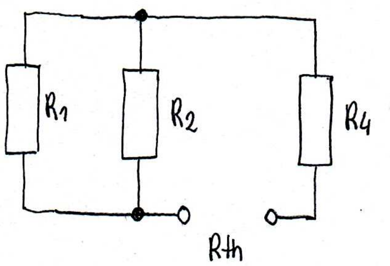
$$I = \frac{U_{th}}{R_{th} + R_5} = \frac{3}{5+1} = \boxed{0.5\,A}$$

$$P_{min} = R_5 \cdot I^2 = 1 \cdot 0.5^2 = \boxed{0.25\,W}$$

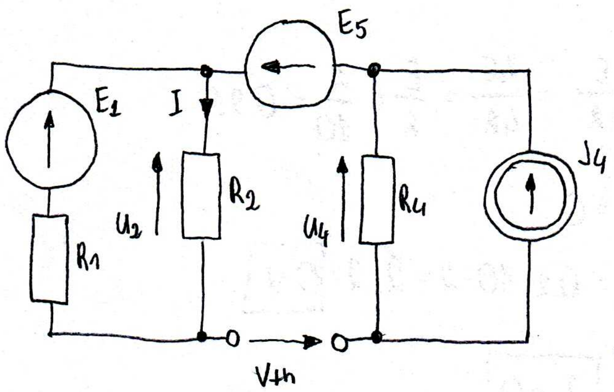
PROBLEM #3

Use Thevenin's theorem to determine the voltage source E_5 that will make the current through R_3 equal to 0A. $R_1 = R_4 = 1\,\Omega$, $R_2 = R_3 = 2\,\Omega$, $J_4 = 1A$, $E_1 = 3V$.





$$R_{th} = \frac{R_1 \cdot R_2}{R_1 + R_2} + R_4 = \frac{1 \cdot 2}{1+2} + 1 = \frac{2}{3} + 1 = \frac{5}{3}\,\Omega$$

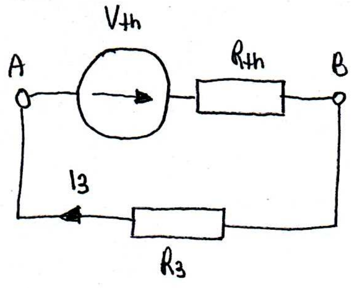


$$I = \frac{E_1}{R_1 + R_2} \qquad U_2 = I \cdot R_2 = \frac{E_1 \cdot R_2}{R_1 + R_2} = \frac{2 \cdot 2}{1+2} = \frac{6}{3} = 2V$$

$$U_4 = R_4 \cdot J_4 = 1 \cdot 1 = 1\,V$$

$$V_{th} + U_4 + E_5 - U_2 = 0$$

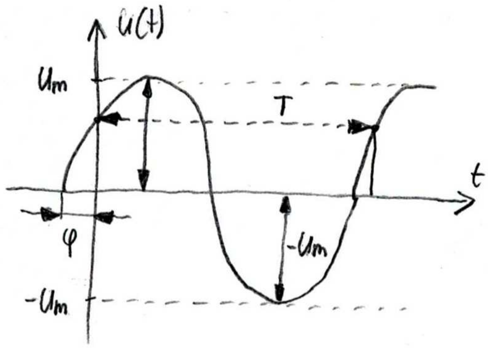
$$V_{th} = U_2 - U_4 - E_5 = 2 - 1 - E_5 = 1 - E_5$$



$$I_3 = \frac{V_{th}}{R_{th} + R_3} = 0 \qquad \frac{1 - E_5}{R_{th} + R_3} = 0 \Rightarrow 1 - E_5 = 0 \rightarrow \boxed{E_5 = 1\,V}$$

A SINE WAVE

$u(t) = U_m \cdot \sin(\omega t + \varphi)$



φ - phase angle or phase shift

$u(t)$ - the instantaneous value of the signal

U_m - the maximum value of the signal
the peak value of the signal (amplitude)
the letter symbol for the peak value of an alternating source voltage is U_m (m for maximum)

T - period [s]

f - frequency [Hz] - the inverse of T, $f = \frac{1}{T}$

ω - angular velocity [rad/s] - radian per second

$\omega = 2\pi f = \frac{2\pi}{T}$

U_{rms} - rms value (root-mean square), the effective value

$U_{rms} = \sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt}$ $\quad U_{rms} = \frac{U_m}{\sqrt{2}}$ the rms value of a sine wave of voltage is $1/\sqrt{2}$ of the peak value

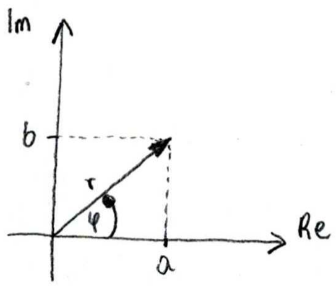
COMPLEX NUMBERS

* complex number can be represented in the form:
  - algebraic form (rectangular form): $\underline{z} = a + jb$
  - trigonometric form (polar form): $\underline{z} = r(\cos\varphi + j\sin\varphi)$
  - exponential form: $\underline{z} = r \cdot e^{j\varphi}$

* <u>algebraic (rectangular) form</u> of complex number

$\underline{z} = a + jb$

- complex numbers have a real part and an imaginary part
- a - real part ($Re(\underline{z})$) of complex number, $Re[z]$
- b - imaginary part of complex number, $Im[z]$
- j - imaginary unit (number) $j^2 = -1$, $j = \sqrt{-1}$ - square root of minus one
- graphical representation of complex number



- we use horizontal axis for the real part and vertical axis for the imaginary part
- $r = |z| = \sqrt{a^2 + b^2}$ - the absolute value of complex number (modulus)
- φ - argument of complex number

  $\varphi = \text{arc tg}\,\frac{b}{a}$

* trigonometric (polar) form of complex number

$$\underline{z} = r\,(\cos\varphi + j\sin\varphi)$$

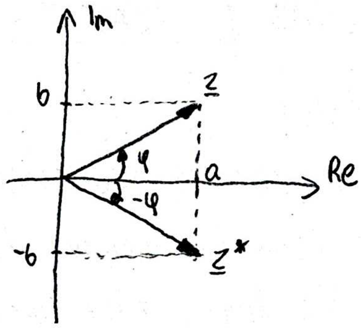
$$r = \sqrt{a^2+b^2} \qquad \varphi = \operatorname{arc\,tg}\frac{b}{a} \qquad \underline{z} = a + jb = r\left(\underbrace{\frac{a}{r}}_{\cos\varphi} + j\underbrace{\frac{b}{r}}_{\sin\varphi}\right) = r\,(\cos\varphi + j\sin\varphi)$$

* exponential form of complex number

$$\underline{z} = re^{j\varphi} \qquad e^{j\varphi} = \cos\varphi + j\sin\varphi$$

* complex conjugate of z

$$\left.\begin{aligned} \underline{z} &= a + jb \\ \underline{z} &= r\,(\cos\varphi + j\sin\varphi) \\ \underline{z} &= re^{j\varphi} \end{aligned}\right\} \Rightarrow \begin{aligned} \underline{z}^* &= a - jb \\ \underline{z}^* &= r\,(\cos\varphi - j\sin\varphi) \\ \underline{z}^* &= re^{-j\varphi} \end{aligned}$$



BASIC OPERATIONS ON COMPLEX NUMBERS

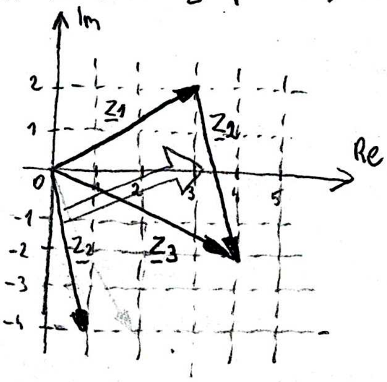
$$\underline{z}_1 = a_1 + jb_1 \qquad \underline{z}_2 = a_2 + jb_2$$

* sum of complex numbers

$$\underline{z} = \underline{z}_1 + \underline{z}_2 = a_1 + jb_1 + a_2 + jb_2 = \underbrace{a_1 + a_2}_{Re} + j\,\underbrace{(b_1 + b_2)}_{Im}$$

we can add complex numbers graphically ['græfɪklɪ]

$$\underline{z}_1 = 3 + j2$$
$$\underline{z}_2 = 1 - j4$$
$$\underline{z}_1 + \underline{z}_2 = 4 - j2$$



o we move vector $\underline{z}_2$ to the end of vector $\underline{z}_1$

* difference of complex numbers

$$\underline{z} = \underline{z}_1 - \underline{z}_2 = \underbrace{a_1 - a_2}_{Re} + j\,\underbrace{(b_1 - b_2)}_{Im}$$

* product of complex number

$$\underline{z} = \underline{z}_1 * \underline{z}_2 = (a_1 + jb_1) * (a_2 + jb_2) = \underbrace{a_1 \cdot a_2 - b_1 \cdot b_2}_{Re} + j\,\underbrace{(a_1 b_2 + b_1 a_2)}_{Im}$$

* division of complex number

$$\underline{z} = \frac{\underline{z}_1}{\underline{z}_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)\cdot(a_2 - jb_2)}{(a_2 + jb_2)\cdot(a_2 - jb_2)} = \underbrace{\frac{a_1 \cdot a_2 + b_1 \cdot b_2}{a_2^2 + b_2^2}}_{Re} + j \underbrace{\frac{b_1 \cdot a_2 - a_1 \cdot b_2}{a_2^2 + b_2^2}}_{Im}$$

- we multiply numerator and denominator by the conjugate of the denominator

PROBLEM #4

Find the complex form (phasor representation) of the following signals.

a) $u_1(t) = 230\sqrt{2}\sin(314 \cdot t)$ V

$$\boxed{u(t) = U_m \cdot \sin(\omega t + \varphi)} \rightarrow \boxed{\underline{U} = U(\cos\varphi + j\sin\varphi)} \quad U_m = 230\sqrt{2} \quad \varphi = 0^\circ \quad U = \frac{U_m}{\sqrt{2}} = \frac{230\sqrt{2}}{\sqrt{2}} = 230$$

$$\underline{U}_1 = 230(\cos 0^\circ + j\sin 0^\circ) = 230(1 + j0) = \boxed{230\,V}$$

b) $i_1(t) = 10\sin(314t - 45^\circ)$ A $\quad \varphi = -45^\circ$

$$\underline{I}_1 = \frac{10}{\sqrt{2}}(\cos(-45^\circ) + j\sin(-45^\circ)) = \frac{10}{\sqrt{2}}\left(\frac{\sqrt{2}}{2} + j\,\frac{-\sqrt{2}}{2}\right) = \frac{10\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\cdot 2} - j\,\frac{10\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\cdot 2} = \boxed{5 - j5\,A}$$

c) $u_2(t) = 25\cos(314 \cdot t)$ V $\quad \cos\varphi = \sin(\varphi + 90^\circ)$

$u_2(t) = 25\sin(314 \cdot t + 90^\circ)$ V

$$\underline{U}_2 = \frac{25}{\sqrt{2}}(\cos 90^\circ + j\sin 90^\circ) = j\frac{25}{\sqrt{2}} = j\,\frac{25\sqrt{2}}{2} = \boxed{j\,17.68\,V}$$

d) $i_2(t) = \cos(314 \cdot t - 30^\circ)$ A $\qquad \cos(\alpha) = \sin(\alpha + 90^\circ)$

$i_2(t) = \sin(314 \cdot t - 30^\circ + 90^\circ)$ A

$i_2(t) = \sin(314 \cdot t + 60^\circ)$ A

$$\underline{I}_2 = \frac{1}{\sqrt{2}}(\cos 60^\circ + j\sin 60^\circ) = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{1}{2\sqrt{2}} + j\,\frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}}{4} + j\frac{\sqrt{6}}{4} = \boxed{(0.353 + j\,0.612)\,A}$$

e) $u_3(t) = 100\sin(t)$ V

$$\underline{U}_3 = \frac{100}{\sqrt{2}}(\cos 0^\circ + j\sin 0^\circ) = \frac{100}{\sqrt{2}} = \boxed{70.71\,V}$$