ELECTRICAL CIRCUITS 1 - CLASS NO. 09 (09.12.2024)

PROBLEM #1

Based on phasor representation, express the voltage and current signals as time functions in the form $f(t) = F_m \sin(\omega t + \varphi)$.

a) $\underline{U}_1 = 100e^{j30^\circ}$ V

$u(t) = U_m \sin(\omega t + \varphi)$ $\quad U_m = 100\sqrt{2}$ $\quad \varphi = 30^\circ$

$$\boxed{u_1(t) = 100\sqrt{2}\sin(\omega t + 30^\circ)\ \text{V}}$$

b) $\underline{I}_1 = e^{j90^\circ}$ A

$I_m = \sqrt{2}$ $\quad \varphi = 90^\circ$

$$\boxed{i_1(t) = \sqrt{2}\sin(\omega t + 90^\circ)}$$

c) $\underline{U}_2 = (20 + j45)$ V

$U_m = \sqrt{20^2 + 45^2}\cdot\sqrt{2} = 49.24\sqrt{2}$

$\varphi = \text{arc tg}\,\frac{45}{20} = 66{,}037^\circ$

$$\boxed{u_2(t) = 49.24\sqrt{2}\sin(\omega t + 66{,}037^\circ)\ \text{V}}$$

d) $\underline{I}_2 = (5 - j5)$ A

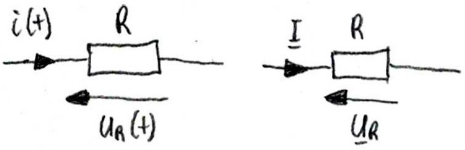
$I_m = \sqrt{2}\sqrt{5^2 + 5^2} = \sqrt{2}\cdot\sqrt{25+25} = \sqrt{100} = 10$

$\varphi = \text{arc tg}\,\frac{-5}{5} = -45^\circ$

$$\boxed{i_2(t) = 10\sin(\omega t - 45^\circ)\ \text{A}}$$

RESISTOR

* Resistor, Resistance [Ω]



$u_R(t) = R\cdot i(t)$

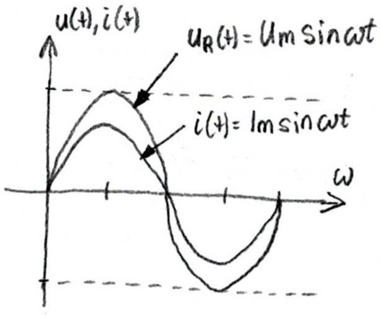
$\underline{U}_R = R\cdot\underline{I}$

R - resistance [Ω]

G - conductance [S]

Z - impedance

Y - admittance



phasor diagram

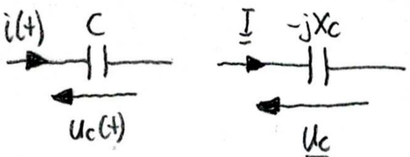
* the voltage across a resistance is <u>in phase</u> with the current

$\psi_u = \psi_i$

$G = \frac{1}{R}$ $\quad \underline{Z} = R$ $\quad \underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R} = G$

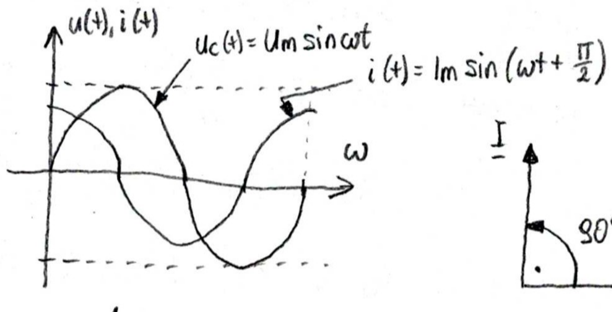
CAPACITOR

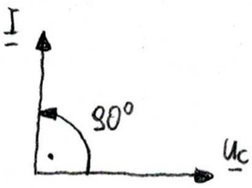
* Capacitor, Capacitance [F]



X_C - capacitive reactance [Ω]

B_C - capacitive susceptance [S]





phasor diagram

$X_C = \frac{1}{\omega C}$ $\quad \omega = 2\pi f$

$\underline{Z} = -jX_C$ $\quad \underline{Y} = \frac{1}{-jX_C} = jB_C$

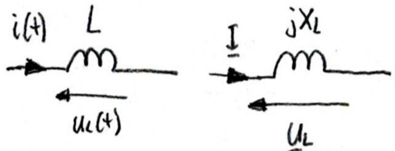
$u_C(t) = \frac{1}{C}\int i(t)\,dt$ $\quad \underline{U}_C = -jX_C\cdot\underline{I}$

* the voltage across a capacitor <u>lags</u> the current by 90° ($\frac{\pi}{2}$)

$\psi_i = \psi_u + 90^\circ$

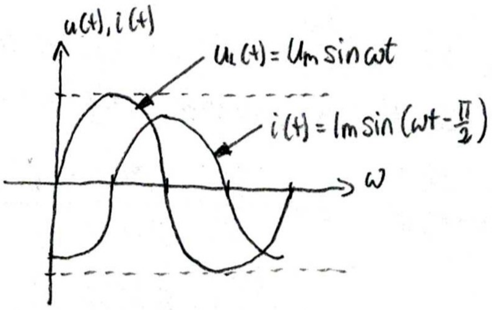
INDUCTOR

* Inductor, Inductance [H]



X_L - inductive reactance [Ω]

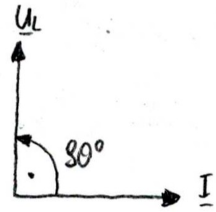
B_L - inductive susceptance [S]



$X_L = \omega L \qquad \omega = 2\pi f$

$\underline{Z} = jX_L \qquad \underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{jX_L} = -jB_L$

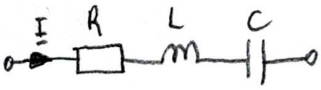
$u_L(t) = L\frac{di(t)}{dt} \qquad \underline{U}_L = jX_L \cdot \underline{I}$



* the voltage across an inductor leads the current by 90° ($\frac{\pi}{2}$)

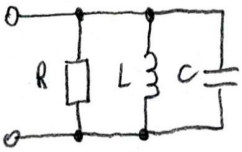
$\varphi_u = \varphi_i + 90°$

IMPEDANCE / ADMITTANCE



$\underline{Z} = R + jX_L - jX_C \qquad \underline{I} = \frac{\underline{U}}{\underline{Z}} = \underline{Y} \cdot \underline{U} \qquad |Z| = \sqrt{R^2 + (X_L - X_C)^2}$

$\underline{Y} = \frac{1}{\underline{Z}} \qquad \varphi = \text{arc tg} \frac{X_L - X_C}{R}$



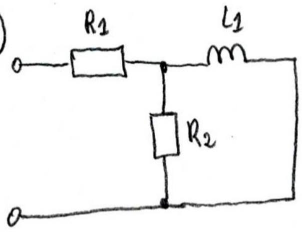
$\underline{Y} = G + jB_C - jB_L$

$\underline{I} = \underline{Y} \cdot \underline{U}$

PROBLEM #2

Find the equivalent impedance and admittance of circuits shown below.

a)



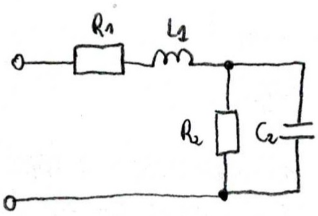
$R_1 = 5Ω$
$R_2 = 20Ω$
$L_1 = 100mH$
$f = 50Hz$

$X_{L1} = 2\pi f L_1 = 2 \cdot 3.14 \cdot 50 \cdot 0.1 = 31.4\ Ω$

$Z_{eq} = R_1 + \frac{R_2 \cdot jX_{L1}}{R_2 + jX_{L1}} = 5 + \frac{20 \cdot j31.4}{20 + j31.4} = \boxed{19.2320 + j9.0604\ Ω}$

$Y_{eq} = 1/Z_{eq} = \boxed{0.0426 - j0.02\ S}$

b)



$R_1 = 25Ω$
$L_1 = 50mH$
$R_2 = 100Ω$
$C_2 = 10\mu F$
$f = 50Hz$

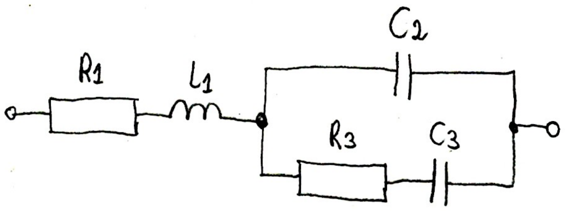
$X_{L1} = 2 \cdot \pi \cdot f \cdot L_1 = 2 \cdot 3.14 \cdot 50 \cdot 0.05 = 15.708\ Ω$

$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2 \cdot 3.14 \cdot 50 \cdot 10 \cdot 10^{-6}} = 318.3099\ Ω$

$Z_{eq} = R_1 + jX_{L1} + \frac{R_2 \cdot -jX_{C2}}{R_2 - jX_{C2}} = 25 + j15.708 + \frac{100 \cdot (-j318.3099)}{100 - j318.3099} =$

$= \boxed{116.017 - j12.886\ Ω} \qquad Y = 1/Z_{eq} = \boxed{0.0085 + j0.009\ S}$

c)



$R_1 = 10\,\Omega$, $R_3 = 20\,\Omega$, $L_1 = 50\,mH$
$C_2 = 2\,mF$, $C_3 = 1\,mF$
$\omega = 100$ rad/s

$X_{L1} = \omega L_1 = 100 \cdot 0.05 = 5\,\Omega$

$X_{C2} = \frac{1}{\omega C_2} = \frac{1}{100 \cdot 0.002} = 5\,\Omega$

$X_{C3} = \frac{1}{\omega C_3} = \frac{1}{100 \cdot 0.001} = 10\,\Omega$

$\underline{Z}_1 = R_1 + jX_{L1} = (10 + j5)\,\Omega$

$\underline{Z}_2 = -jX_{C2} = -j5\,\Omega$

$\underline{Z}_3 = R_3 - jX_{C3} = (20 - j10)\,\Omega$

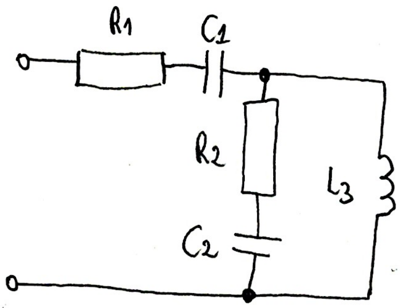
$\underline{Z}_2 = \underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}$

$\underline{Z}_2 = 10 + j5 + \frac{(-j5)(20-j10)}{-j5+20-j10}$

$\underline{Z}_2 = 10 + j5 + 0.8 - j\,4.4$

$\underline{Z}_2 = \boxed{(10.8 + j\,0.6)\,\Omega}$

d)



$R_1 = 10\,\Omega$, $R_2 = 5\,\Omega$, $X_{C1} = 20\,\Omega$, $X_{C2} = 10\,\Omega$, $X_{L3} = 20\,\Omega$

$\underline{Z}_1 = R_1 - jX_{C1} = (10 - j20)\,\Omega$

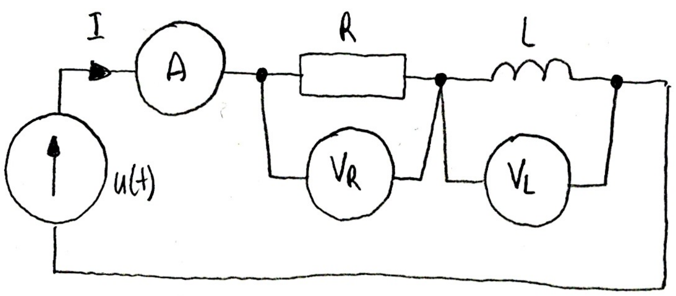
$\underline{Z}_2 = R_2 - jX_{C2} = (5 - j10)\,\Omega$

$\underline{Z}_3 = jX_{L3} = j20\,\Omega$

$\underline{Z}_{eq} = \underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = (10 - j20) + \frac{(5-j10)(j20)}{5-j10+j20} = 10 - j20 + 16 - j12 = \boxed{(26 - j32)\,\Omega}$

PROBLEM #3

Calculate the meter readings in the circuit shown in the figure.
$u(t) = 230\sqrt{2}\sin\omega t$ V, $R = 40\,\Omega$, $L = 0.2$ H, $f = 50$ Hz.



$\omega = 2\pi f$

$X_L = \omega L = 2\pi f L = 2 \cdot 3.14 \cdot 50 \cdot 0.2 = 62.8319\,\Omega$

Method 1

$Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 62.8319^2} = 74.4838\,\Omega$

$I = \frac{U}{Z} = \frac{230}{74.4838} = 3.0879$ A $\quad I_A = I = \boxed{3.0879\text{ A}}$

$U_R = R \cdot I = 40 \cdot 3.0879 = 123.5167$ V $\quad V_R = U_R = \boxed{123.5167\text{ V}}$

$U_L = X_L \cdot I = 62.8319 \cdot 3.0879 = 194.0186$ V $\quad V_L = U_L = \boxed{194.0186\text{ V}}$

Method 2

$u(t) = U_m \sin(\omega t + \varphi)$ $\quad$ $\underline{U} = U_{rms} \cdot (\cos\varphi + j\sin\varphi)$ $\quad$ $U_{rms} = \frac{U_m}{\sqrt{2}}$

$U_{rms} = \frac{230\sqrt{2}}{\sqrt{2}} = 230$ $\quad$ $\varphi = 0°$

$\underline{U} = 230(\cos 0° + j\sin 0°) = 230(1 + j0) = 230\ V$

$\underline{Z} = R + jX_L = (40 + j62.8319)\,\Omega$

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{230}{40 + j62.8319} = (1.6583 - j2.6049)\,A$ $\quad$ $|\underline{I}| = \sqrt{1.6583^2 + 2.6049^2} = 3.0879\ A$

$I_A = |\underline{I}| = \boxed{3.0879\ A}$

$\underline{U}_R = R \cdot \underline{I} = 40 \cdot (1.6583 - j2.6049) = (66.3321 - j104.1942)\,V$ $\quad$ $V_R = |\underline{U}_R| = \boxed{123.5167\ V}$

$\underline{U}_L = jX_L \cdot \underline{I} = j62.8319 \cdot (1.6583 - j2.6049) = (163.6679 + j104.1942)\,V$ $\quad$ $V_L = |\underline{U}_L| = \boxed{194.0196\ V}$

Phasor diagram

$\underline{U} = 230 = 230e^{j0°}\,V$

$\underline{U}_R = 66.3321 - j104.1942 = 123.52e^{-j57.52°}\,V$

$\underline{U}_L = 163.6679 + j104.1942 = 194.02e^{j32.48°}\,V$

$\underline{I} = 1.6583 - j2.6049 = 3.08e^{-j57.52°}\,A$

$\boxed{\underline{U} = \underline{U}_R + \underline{U}_L}$