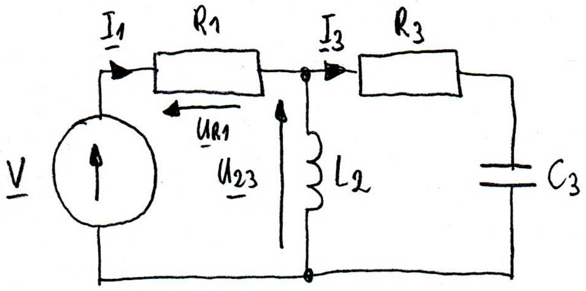
ELECTRICAL CIRCUITS 1 - CLASS NO. 10 (09.01.2025)

PROBLEM #1

In the circuit as shown in the figure, the resistor R_3 has the maximum power of $P_3 = 8W$. Check if this is sufficient for the correct operation of the system.
$\underline{V} = 24\angle 60^\circ$ V, $R_1 = 4\Omega$, $X_{L2} = 6\Omega$, $R_3 = 8\Omega$, $X_{C3} = 4\Omega$.



$\underline{V} = V(\cos 60^\circ + j\sin 60^\circ) = 24\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) =$
$= 12 + j12\sqrt{3} = (12 + j\,20.78)\,V$

$\underline{Z}_1 = R_1 = 4\Omega \qquad \underline{Z}_3 = R_3 - jX_{C3} = (8 - j4)\Omega$

$\underline{Z}_2 = jX_{L2} = j6\Omega$

$$\underline{Z}_{23} = \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{j6(8-j4)}{j6+8-j4} = (4.24 + j4.94)\Omega \qquad \underline{Z} = \underline{Z}_1 + \underline{Z}_{23} = 4 + 4.24 + j4.94 = (8.24 + j4.94)\Omega$$

$$\underline{I}_1 = \frac{\underline{V}}{\underline{Z}} = \frac{12 + j20.78}{8.24 + j4.94} = (2.18 + j\,1.21)A$$

$\underline{U}_{23} = ?$ KVL: $\underline{U}_{R1} + \underline{U}_{23} = \underline{V} \Rightarrow \underline{U}_{23} = \underline{V} - \underline{U}_{R1} = \underline{V} - R_1 \cdot \underline{I}_1 = (12 + j20.78) - 4\cdot(2.18 + j\,1.21) =$
$= (3.26 + j\,15.93)V$

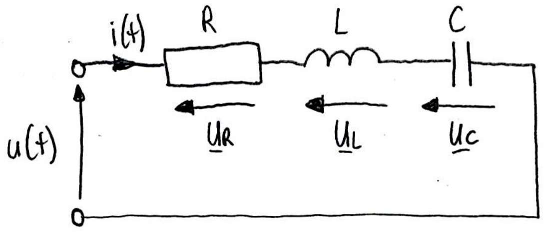
Ohm's Law: $\underline{U}_{23} = \underline{Z}_{23} \cdot \underline{I}_1 = (4.24 + j4.94)(2.18 + j\,1.21) = (3.26 + j\,15.93)\,V$

$$\underline{I}_3 = \frac{\underline{U}_{23}}{\underline{Z}_3} = \frac{3.26 + j\,15.93}{8 - j4} = (-0.47 + j\,1.76)\,A \qquad |\underline{I}_3| = \sqrt{(-0.47)^2 + (1.76)^2} = 1.82$$

$$P_3 = R_3 \cdot I_3^2 = 8 \cdot 1.82 = \boxed{26.45\,W} > 8W$$

PROBLEM #2

Calculate the current $i(t)$ and the voltage drops $\underline{U}_R$, $\underline{U}_L$, and $\underline{U}_C$ in the circuit shown below. Draw a phasor diagram for this circuit. $R = 20\Omega$, $L = 20mH$, $C = 100\mu F$, $u(t) = 100\sqrt{2}\cos\omega t$ V, $\omega = 10^3$ rad/s.



$X_L = \omega L = 1000 \cdot 0.02 = 20\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \cdot 100 \cdot 10^{-6}} = 10\Omega$$

$\underline{Z} = R + jX_L - jX_C = 20 + j20 - j10 = (20 + j10)\Omega$

Method 1

$u(t) = U_m \cos(\omega t + \varphi)$ $\quad U_m = 100\sqrt{2} \quad \varphi = 0° \quad U_{rms} = \frac{U_m}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$

$$\underline{U} = U_{rms}(\cos\varphi + j\sin\varphi) = 100(\cos 0° + j\sin 0°) = 100(1 + j0) = 100 \text{ V}$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{100}{20+j10} = \frac{100(20-j10)}{(20+j10)(20-j10)} = \frac{200-j100}{20^2+10^2} = \frac{2000-j1000}{500} = (4-j2)\text{ A} = 4.47e^{-j26.57°}\text{ A}$$

$$\underline{U_R} = R\cdot\underline{I} = 20(4-j2) = (80-j40)\text{ V} = 89.44e^{-j26.57°}\text{ V}$$

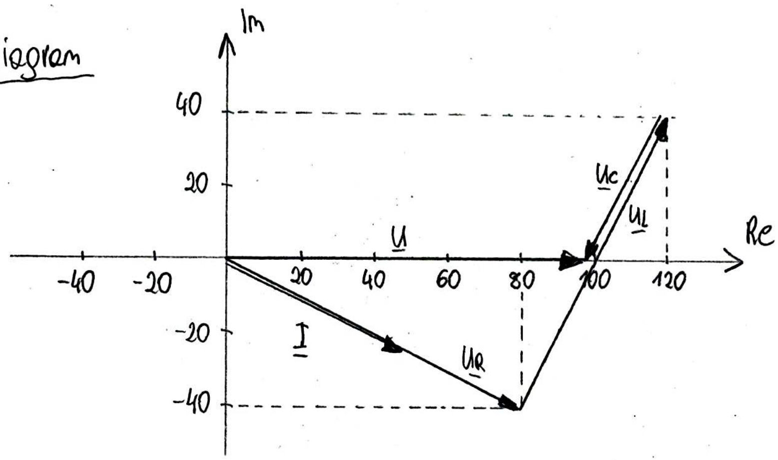
$$\underline{U_L} = jX_L\cdot\underline{I} = j20(4-j2) = (40+j80)\text{ V} = 89.44e^{j63.44°}\text{ V}$$

$$\underline{U_C} = -jX_C\cdot\underline{I} = -j10(4-j2) = (-20-j40)\text{ V} = 44.72e^{-j116.57°}\text{ V}$$

$$\boxed{i(t) = 4.47\sqrt{2}\cos(\omega t - j26.57°)\text{ A}} \Rightarrow \boxed{i(t) = 4.47\sqrt{2}\sin(\omega t + 63.44°)\text{ A}}$$

$\cos\alpha = \sin(\alpha + 90°)$

Phasor diagram



$$U_{Rrms} + U_{Lrms} + U_{Crms} \neq U_{rms}$$

$$89.44 + 89.44 + 44.72 \neq 100$$

$$226.6 \neq 100$$

$$\underline{U_R} + \underline{U_L} + \underline{U_C} = \underline{U}$$

$$80 - \cancel{j40} + 40 + \cancel{j80} - 20 - \cancel{j40} = 100$$

$$100 = 100$$

Method 2

$\cos\alpha = \sin(\alpha + 90°)$

$u(t) = U_m\cos(\omega t + \varphi) = U_m\sin(\omega t + \varphi + 90°)$

$u(t) = 100\sqrt{2}\cos\omega t = 100\sqrt{2}\sin(\omega t + 90°) \quad U_{rms} = 100, \quad \varphi = 90°$

$\underline{U} = 100(\cos 90° + j\sin 90°) = 100(0 + j1) = j100\text{ V}$

$\underline{Z} = (20 + j10)\ \Omega$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{j100}{20+j10} = \frac{j100(20-j10)}{(20+j10)(20-j10)} = \frac{1000+j2000}{20^2+10^2} = \frac{1000+j2000}{500} = (2+j4)\text{ A} = 4.47e^{j63.44°}\text{ A}$$

$$\boxed{i(t) = 4.47\sqrt{2}\sin(\omega t + 63.44°)\text{ A}}$$

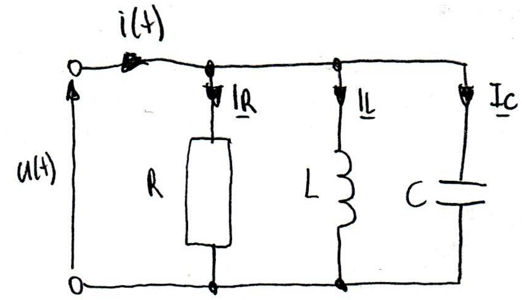
$$\underline{U_R} = R\cdot\underline{I} = 20(2+j4) = (40+j80)\text{ V} = 89.44e^{j63.44°}\text{ V}$$

$$\underline{U_L} = jX_L\cdot\underline{I} = j20(2+j4) = (-80+j40)\text{ V} = 89.44e^{j153.44°}\text{ V}$$

$$\underline{U_C} = -jX_C\cdot\underline{I} = -j10(2+j4) = (40-j20)\text{ V} = 44.72e^{-j26.57°}\text{ V}$$

PROBLEM #3

Calculate the currents $i(t)$, $\underline{I}_R$, $\underline{I}_L$, and $\underline{I}_C$ in the circuit shown below. Draw a phasor diagram for this circuit. $R = 20\,\Omega$, $L = 20\,\text{mH}$, $C = 100\,\mu\text{F}$, $u(t) = 100\sqrt{2}\cos\omega t$ V, $\omega = 10^3$ rad/s.



$$X_L = \omega L = 1000 \cdot 0.02 = 20\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \cdot 100 \cdot 10^{-6}} = 10\,\Omega$$

$$\underline{U} = 100\,(\cos 0^\circ + j\sin 0^\circ) = 100\text{ V}$$

$$G = \frac{1}{R} = \frac{1}{20} = 0.05\,\Omega^{-1}$$

$$B_C = \omega C = 1000 \cdot 100 \cdot 10^{-6} = 0.1\,\Omega^{-1}$$

$$B_L = \frac{1}{\omega L} = \frac{1}{1000 \cdot 0.02} = 0.05\,\Omega^{-1}$$

Method 1

$$\underline{Y} = G + j(B_C - B_L)$$

$$\underline{Y} = 0.05 + j(0.1 - 0.05)\,\Omega^{-1}$$

$$\underline{Y} = (0.05 + j\,0.05)\,\Omega^{-1}$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \underline{U} \cdot \underline{Y} = 100 \cdot (0.05 + j0.05) = (5 + j5)\text{A} = 7.07\,e^{j45^\circ}\text{A}$$

$$\boxed{i(t) = 7.07\sqrt{2}\cos(\omega t + 45^\circ)\text{ A}}$$

Method 2

$$\underline{I}_R = \frac{\underline{U}}{R} = \frac{100}{20} = \boxed{5\text{ A}} = 5e^{j0^\circ}\text{A}$$

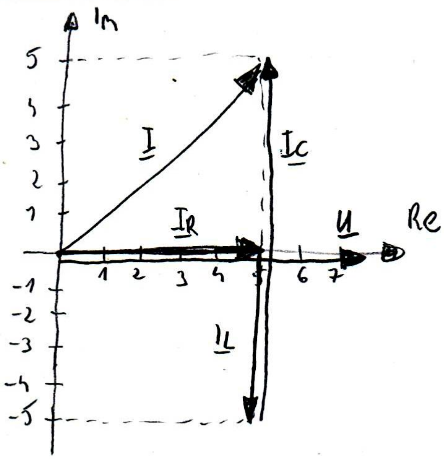
$$\underline{I}_L = \frac{\underline{U}}{jX_L} = \frac{100}{j20} = \boxed{-j5\text{ A}} = 5e^{-j90^\circ}\text{A}$$

$$\underline{I}_C = \frac{\underline{U}}{-jX_C} = \frac{100}{-j10} = \boxed{j10\text{ A}} = 10e^{j90^\circ}$$

$$\underline{I} = \underline{I}_R + \underline{I}_L + \underline{I}_C = 5 - j5 + j10 = (5 + j5)\text{A} = 7.07\,e^{j45^\circ}\text{A}$$

$$\boxed{i(t) = 7.07\sqrt{2}\cos(\omega t + 45^\circ)\text{A}}$$

Phasor diagram



$$I_{R\,rms} + I_{L\,rms} + I_{C\,rms} \neq I_{rms}$$

$$5 + 5 + 10 \neq 7.07$$

$$20 \neq 7.07$$

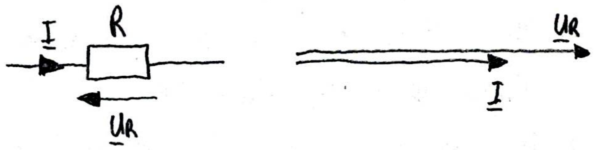
$$\underline{I}_R + \underline{I}_L + \underline{I}_C = \underline{I}$$

$$5 - j5 + j10 = 5 + j5$$

$$5 + j5 = 5 + j5$$

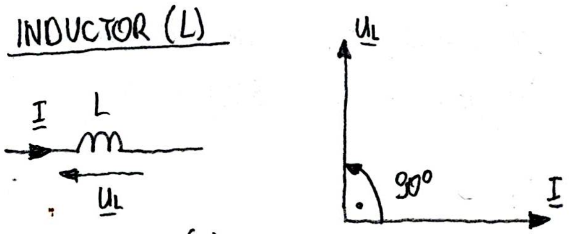
PHASOR DIAGRAMS

* RESISTOR (R)



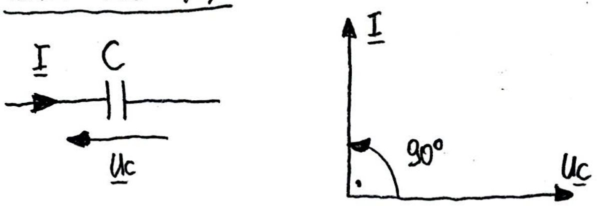
* the voltage U_R across a resistance R is in phase with the current I

* INDUCTOR (L)



* the voltage U_L across an inductor L <u>leads</u> the current I by 90° ($\frac{\pi}{2}$)

* CAPACITOR (C)

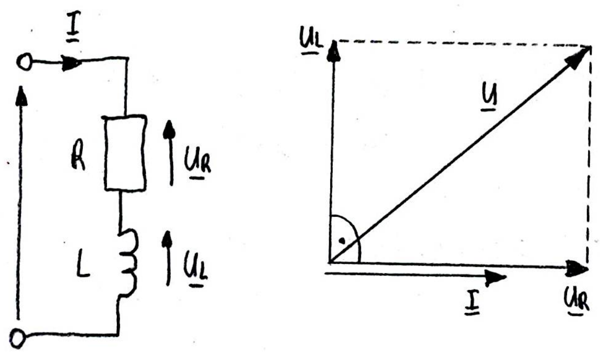


* the voltage U_C across a capacitor C <u>lags</u> the current I by 90° ($\frac{\pi}{2}$)

PROBLEM #4

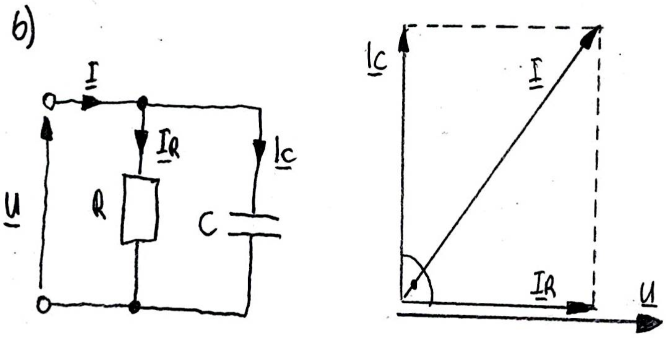
Draw phasor diagrams for the following circuits.

a)



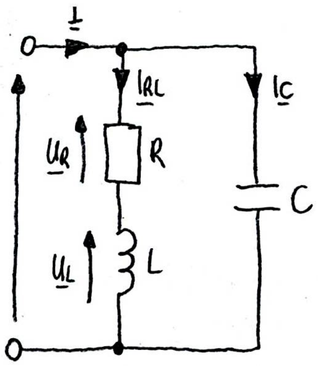
* we start from the current I and we assume a phase angle of 0°
* the voltage U_R across a resistance R is in phase with the current I
* the voltage U_L across an inductor L leads the current I by 90°
* the voltage U is a sum of voltage drops U_R and U_L (from KVL)

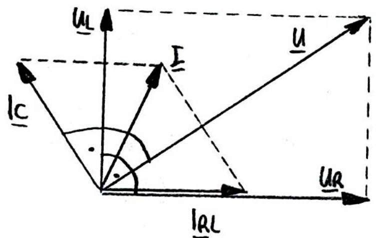
b)



* we start from the current I_R and we assume a phase angle of 0°
* the voltage U is in phase with the current I_R
* the current I_C through a capacitor C leads the current I_R by 90°
* the current I is a sum of currents I_R and I_C (from KCL)

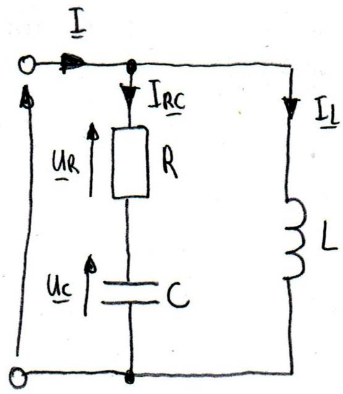
c).

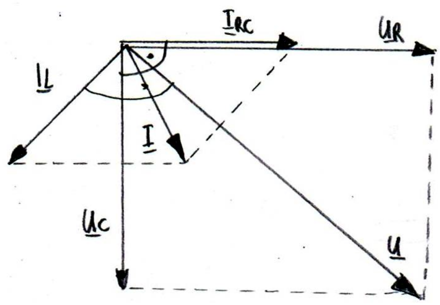




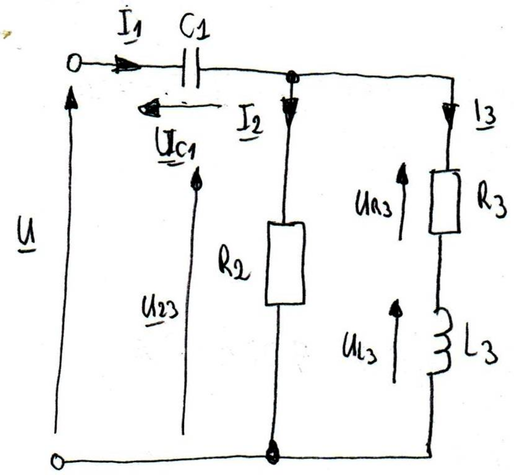
* we start from the current $\underline{I}_{RL}$ and we assume a phase angle of 0°
* the voltage $\underline{U}_R$ is in phase with the cullent $\underline{I}_{RL}$
* the voltage drop $\underline{U}_L$ leads the current $\underline{I}_{RL}$ by 90°
* the voltage $\underline{U}$ is a sum of voltage drops $\underline{U}_R$ and $\underline{U}_L$ (from KVL)
* the current $\underline{I}_C$ leads the voltage $\underline{U}$ by 90°
* the current $\underline{I}$ is a sum of currents $\underline{I}_{RC}$ and $\underline{I}_C$ (from KCL)

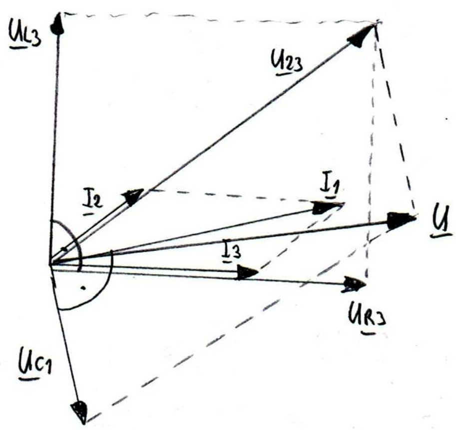
d).





e)





POWER IN AC CIRCUITS

* P - ACTIVE POWER (TRUE POWER, REAL POWER) unit: Watt [W]

$P = U_{rms} \cdot I_{rms} \cdot \cos\varphi$, where $\varphi = \psi_u - \psi_i$ (angle between voltage and current)

$$U_{rms} = \frac{U_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} \quad \rightarrow \quad P = \frac{U_m \cdot I_m}{2} \cdot \cos\varphi \qquad P = R \cdot I_{rms}^2 = \frac{U_{rms}^2}{R}$$

* Q - REACTIVE POWER unit: voltampere (reactive) [var]

$Q = U_{rms} \cdot I_{rms} \cdot \sin\varphi$ $\qquad \varphi = \psi_u - \psi_i$ (angle between voltage and current)

$$Q = \frac{U_m \cdot I_m}{2} \sin\varphi \qquad Q = X \cdot I_{rms}^2 = \frac{U_{rms}^2}{X}$$

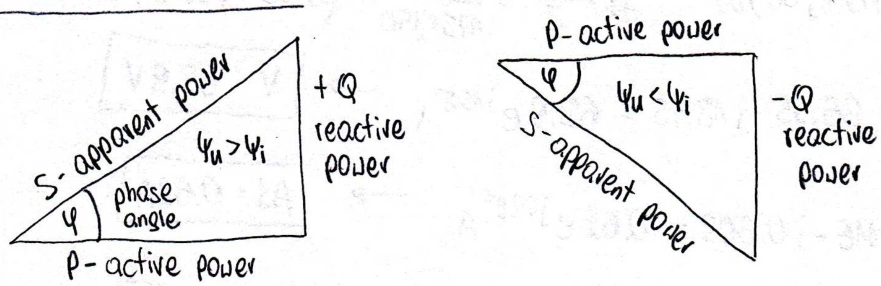
* S - APPARENT POWER unit: voltampere [VA]

$$S = U_{rms} \cdot I_{rms} \qquad S = I_{rms}^2 \cdot Z = \frac{U_{rms}^2}{Z}$$

* $\underline{S}$ - PHASOR POWER (COMPLEX POWER)

$$\underline{S} = \underline{U} \cdot \underline{I}^* \qquad \underline{S} = P + jQ \qquad P = \mathrm{Re}[\underline{S}] \quad Q = \mathrm{Im}[\underline{S}]$$

* THE POWER TRIANGLE

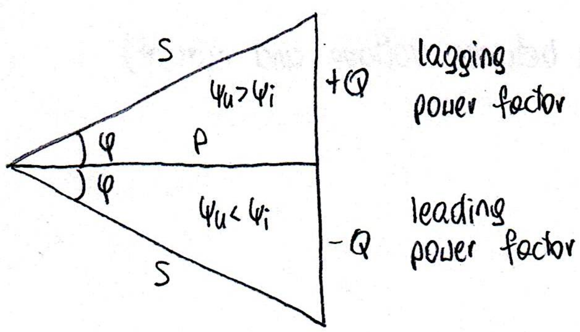


$$S = \sqrt{P^2 + Q^2}$$

* THE POWER FACTOR (pf)

$$(pf) = \frac{P}{S} = \cos\varphi$$

* the ratio between the active power and the apparent power of a load is called the power factor of the load

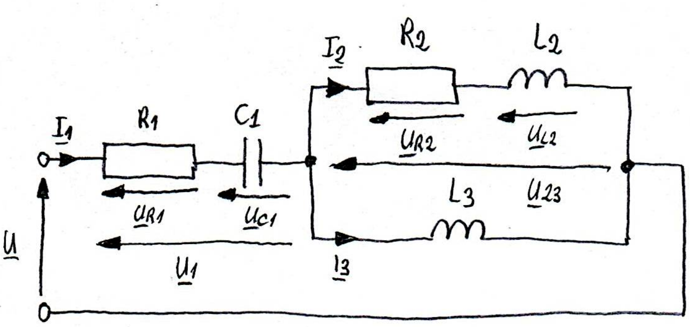


* inductive loads always have a lagging power factor

* capacitive loads always have a leading power factor

PROBLEM #5

Calculathe all currents and voltages in the circuit shown in the figure. Determine the active, reactive, and apparent power consumed by the circuit. Draw a phasor diagram for the circuit.
$\underline{U} = (10+j10)V$, $R_1 = 1\Omega$, $R_2 = 5\Omega$, $X_{C1} = X_{L2} = X_{L3} = 5\Omega$.



$$\underline{Z} = R_1 - jX_{C1} + \frac{(R_2+jX_{L2})\cdot(jX_{L3})}{R_2+jX_{L2}+jX_{L3}} =$$

$$= 1 - j5 + \frac{(5+j5)(j5)}{5+j5+j5} = (2-j2)\,\Omega$$

$$\underline{I}_1 = \frac{\underline{U}}{\underline{Z}} = \frac{10+j10}{2-j2} = \boxed{j5\ A}$$

$$\underline{U}_1 = (R_1 - jX_{C1})\cdot \underline{I}_1 = (1-j5)\cdot(j5) = \boxed{(25+j5)\ V}$$

$$\underline{U}_{23} = \underline{U} - \underline{U}_1 = 10+j10-25-j5 = \boxed{(-15+j5)\ V}$$

$$\underline{I}_2 = \frac{\underline{U}_{23}}{R_2+jX_{L2}} = \frac{-15+j5}{5+j5} = \boxed{(-1+j2)\ A}$$

$$\underline{I}_3 = \frac{\underline{U}_{23}}{jX_{L3}} = \frac{-15+j5}{j5} = \boxed{(1+j3)\ A}$$

$$\underline{U}_{R2} = R_2\cdot\underline{I}_2 = 5\cdot(-1+j2) = \boxed{(-5+j10)\ V}$$

$$\underline{U}_{R1} = R_1\cdot\underline{I}_1 = 1\cdot j5 = \boxed{j5\ V}$$

$$\underline{U}_{L2} = jX_{L2}\cdot\underline{I}_2 = j5\cdot(-1+j2) = \boxed{(-10-j5)\ V}$$

$$\underline{U}_{C1} = (-jX_{C1})\cdot\underline{I}_1 = -j5\cdot j5 = \boxed{25\ V}$$

$$\underline{S} = \underline{U}\cdot\underline{I}_1^* = (10+j10)\cdot(-j5) = \boxed{(50-j50)\ VA}$$

$P = \boxed{50\ W}$ $\quad Q = \boxed{-50\ var}$ $\quad S = \boxed{70.71\ VA}$