ELECTRICAL CIRCUITS 2 - CLASS NO. 2 (11.03.2025)

SELF INDUCTANCE

inductance - L

the unit of inductance : henry

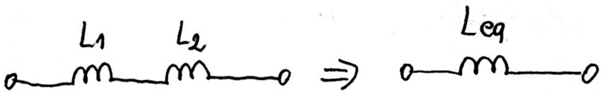
the unit symbol : H

X_L - inductive reactance $\underline{Z} = jX_L = j\omega L$

$X_L = \omega L$, $\omega = 2\pi f$ ω - angular velocity

the unit of inductive reactance : ohm $[\Omega]$

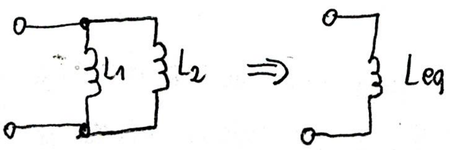
INDUCTORS IN SERIES



$L_{eq} = L_1 + L_2$

when there is no mutual coupling between inductors in series, the total inductance is the sum of the individual inductances

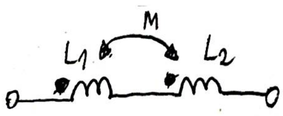
INDUCTORS IN PARALLEL



$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

MUTUAL INDUCTANCE

mutual inductance occurs when inductors are placed in close proximity to one another and share a common magnetic flux



X_M - mutual reactance

$X_M = \omega M$ $\omega = 2\pi f$

the unit of mutual reactance : ohm $[\Omega]$

$\underline{Z} = jX_M = j\omega M$

the letter symbol of mutual inductance : M

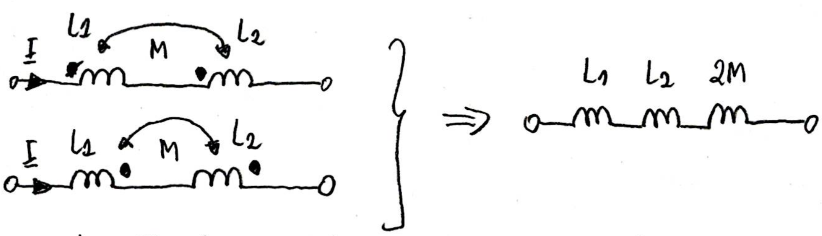
the unit of mutual inductance : henry

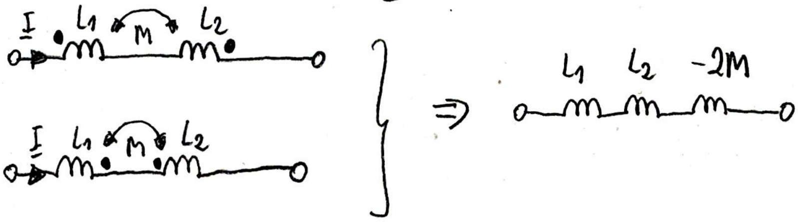
the unit symbol : H

$M = k \cdot \sqrt{L_1 \cdot L_2}$ k - coefficient of coupling $(0 \leq k \leq 1)$

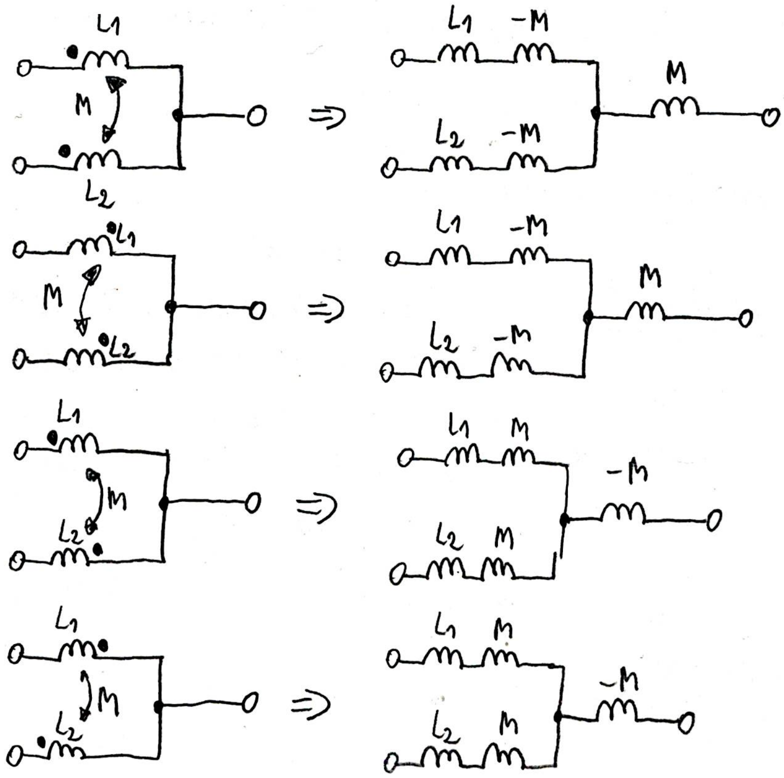
* the coefficient k is an indication of how much flux in one coil is linked with the other coil
* if all the flux in one coil reaches the other coil, then we have 100% coupling and k=1
* if there is no coupling k=0

INDUCTORS IN SERIES WITH MUTUAL INDUCTANCE





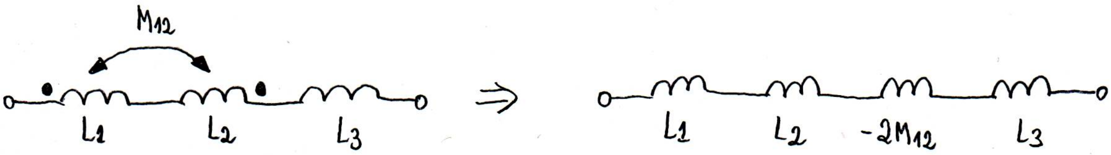
ELIMINATION OF COUPLING IN ELEMENTS HAVING A COMMON NODE



PROBLEM #1

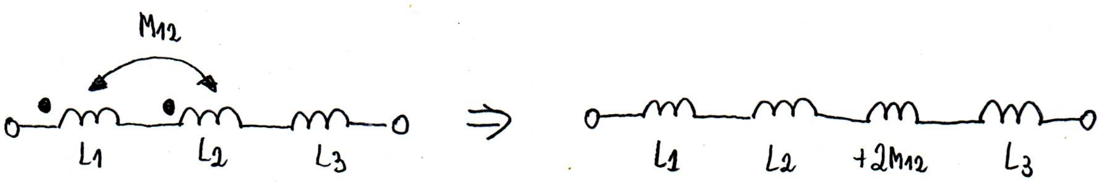
Eliminate couplings and calculate the equivalent inductance of the circuits shown in the figures. $L_1 = 0.1\,H$, $L_2 = 0.2\,H$, $L_3 = 0.4\,H$, $M_{12} = 0.1\,H$, $M_{23} = 0.25\,H$, $M_{13} = 0.2\,H$.

a)



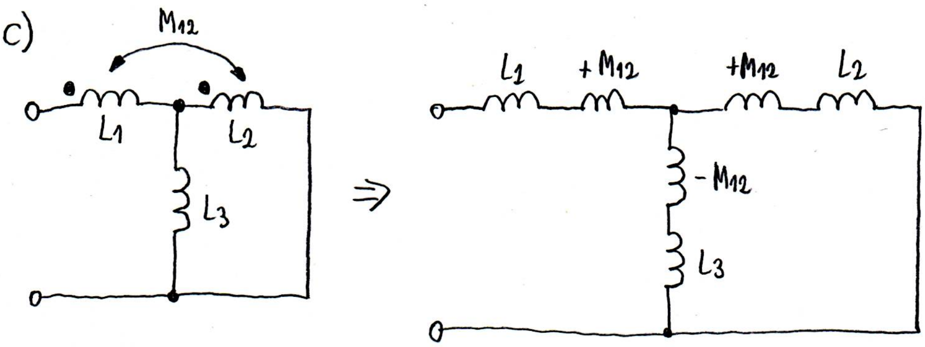
$$L_{eq} = L_1 + L_2 - 2M_{12} + L_3 = 0.1 + 0.2 - 2\cdot 0.1 + 0.4 = \boxed{0.5\,H}$$

b)



$$L_{eq} = L_1 + L_2 + 2\cdot M_{12} + L_3 = 0.1 + 0.2 + 2\cdot 0.1 + 0.4 = \boxed{0.9\,H}$$

c)



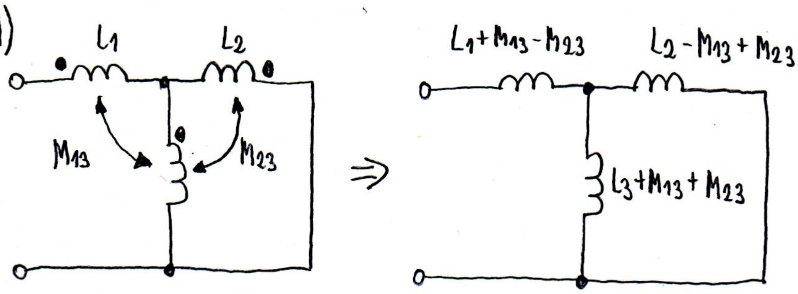
$L_{eq1} = L_1 + M_{12} = 0.1 + 0.1 = 0.2\,H$

$L_{eq2} = L_2 + M_{12} = 0.2 + 0.1 = 0.3\,H$

$L_{eq3} = L_3 - M_{12} = 0.4 - 0.1 = 0.3\,H$

$$L_{eq} = L_{eq1} + \frac{L_{eq2} \cdot L_{eq3}}{L_{eq2} + L_{eq3}} = 0.2 + \frac{0.3 \cdot 0.3}{0.3 + 0.3} = 0.2 + 0.15 = \boxed{0.35\,H}$$

d)



$L_{eq1} = L_1 + M_{13} - M_{23} = 0.1 + 0.2 - 0.25 = 0.05\,H$

$L_{eq2} = L_2 - M_{13} + M_{23} = 0.2 - 0.2 + 0.25 = 0.25\,H$

$L_{eq3} = L_3 + M_{13} + M_{23} = 0.4 + 0.2 + 0.25 = 0.85\,H$

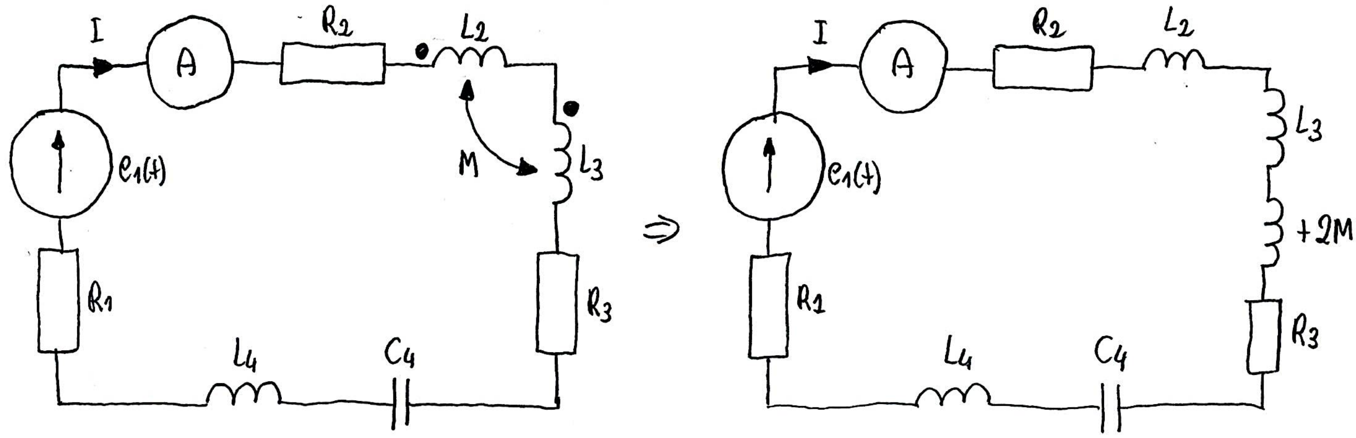
$$L_{eq} = L_{eq1} + \frac{L_{eq2} \cdot L_{eq3}}{L_{eq2} + L_{eq3}} = 0.05 + \frac{0.25 \cdot 0.85}{0.25 + 0.85} = 0.05 + \frac{0.2125}{1.1} = 0.05 + 0.1932 = \boxed{0.2432\,H}$$

PROBLEM #2

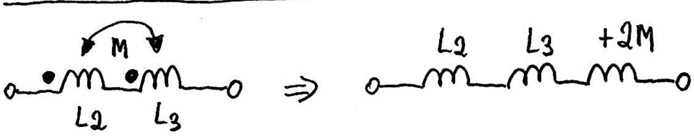
Calculate ammeter readings in the circuits shown in the figures. $M = 0.1\,H$

$e_1(t) = 100\sqrt{2}\sin(100t + 90^\circ)\,V$, $R_1 = 15\,\Omega$, $R_2 = 30\,\Omega$, $L_2 = 0.1\,H$, $L_3 = 0.4\,H$, $R_3 = 15\,\Omega$, $C_4 = 500\,\mu F$, $L_4 = 0.3\,H$.

a)



Elimination of coupling



$\underline{E}_1 = 100(\cos 90^\circ + j\sin 90^\circ) = j100\,V$

$X_{L2} = \omega L_2 = 100 \cdot 0.1 = 10\,\Omega$

$X_{L3} = \omega L_3 = 100 \cdot 0.4 = 40\,\Omega$

$X_M = \omega M = 100 \cdot 0.1 = 10\,\Omega$

$X_{L4} = \omega L_4 = 100 \cdot 0.3 = 30\,\Omega$

$X_{C4} = \frac{1}{\omega C_4} = \frac{1}{100 \cdot 500 \cdot 10^{-6}} = 20\,\Omega$

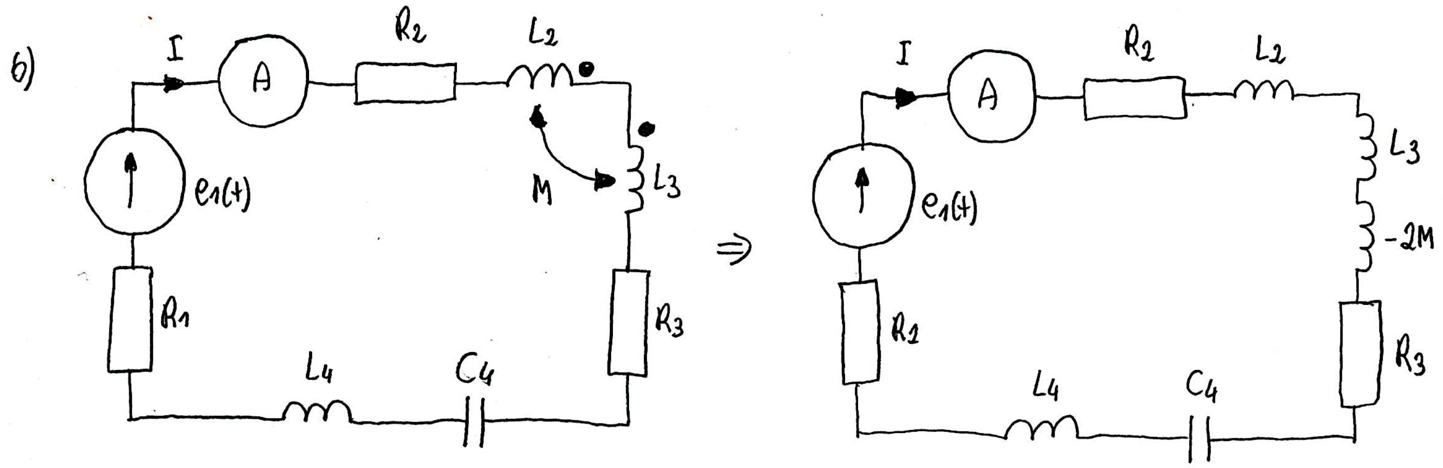
$\underline{Z} = R_2 + jX_{L2} + jX_{L3} + 2jX_M + R_3 + jX_{L4} - jX_{C4} + R_1$

$\underline{Z} = 30 + j10 + j40 + j20 + 15 + j30 - j20 + 15 = (60 + j80)\,\Omega$

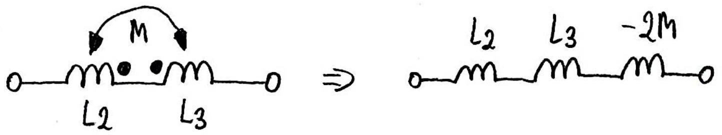
$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{j100}{60 + j80} = (0.8 + j0.6)\,A$

$I_A = |\underline{I}| = \sqrt{0.8^2 + 0.6^2} = \boxed{1\,A}$

b)



Elimination of coupling



$$\underline{Z} = R_1 + R_2 + jX_{L2} + jX_{L3} - j2X_M + R_3 - jX_{C4} + jX_{L4} = 15 + 30 + j10 + j40 - j20 + 15 - j20 + j30 = (60 + j40)\,\Omega$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{j100}{60+j40} = (0.7692 + j\,1.1538)\ A \qquad I_A = |\underline{I}| = \sqrt{0.7692^2 + 1.1538^2} = \boxed{1.39\ A}$$