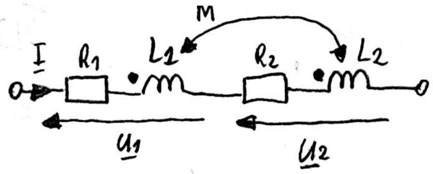
ELECTRICAL CIRCUITS 2 - CLASS NO. 3 (18.03.2025)

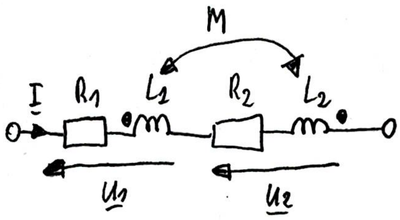
INDUCTORS IN SERIES WITH MUTUAL COUPLING



$$\underline{U}_1 = R_1\underline{I} + j\omega L_1\underline{I} + \boxed{j\omega M}\,\underline{I}$$
$$\underline{U}_2 = R_2\underline{I} + j\omega L_2\underline{I} + \boxed{j\omega M}\,\underline{I}$$

→ $+j\omega M$ when current enters the dots in the both inductors

$$\underline{U} = \underline{U}_1 + \underline{U}_2 = (R_1 + R_2 + j\omega L_1 + j\omega L_2 + j2\omega M)\,\underline{I}$$

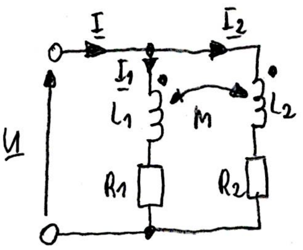


$$\underline{U}_1 = R_1\underline{I} + j\omega L_1\underline{I}\boxed{-j\omega M}\,\underline{I}$$
$$\underline{U}_2 = R_2\underline{I} + j\omega L_2\underline{I}\boxed{-j\omega M}\,\underline{I}$$

→ $-j\omega M$ when current enters a dot in one inductor and leaves a dot in other inductor

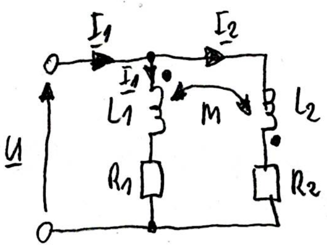
$$\underline{U} = \underline{U}_1 + \underline{U}_2 = (R_1 + R_2 + j\omega L_1 + j\omega L_2 - j2\omega M)\,\underline{I}$$

INDUCTORS IN PARALLEL WITH MUTUAL COUPLING



$$\begin{cases} \underline{I} = \underline{I}_1 + \underline{I}_2 \\ \underline{U} = R_1\underline{I}_1 + j\omega L_1\underline{I}_1 \boxed{+j\omega M}\,\underline{I}_2 \\ \underline{U} = R_2\underline{I}_2 + j\omega L_2\underline{I}_2 \boxed{+j\omega M}\,\underline{I}_1 \end{cases}$$

→ $+j\omega M$ when both currents enter the dots

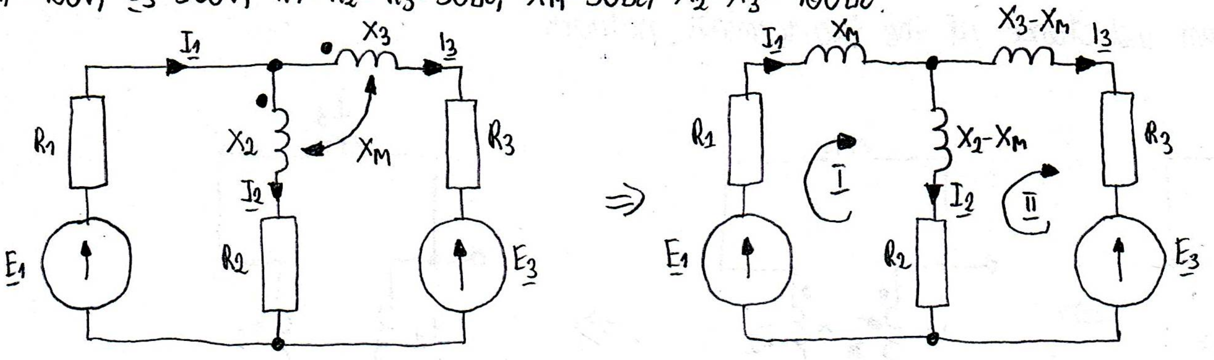


$$\begin{cases} \underline{I} = \underline{I}_1 + \underline{I}_2 \\ \underline{U} = R_1\underline{I}_1 + j\omega L_1\underline{I}_1 \boxed{-j\omega M}\,\underline{I}_2 \\ \underline{U} = R_2\underline{I}_2 + j\omega L_2\underline{I}_2 \boxed{-j\omega M}\,\underline{I}_2 \end{cases}$$

→ $-j\omega M$ when one current enters a dot and the other current leaves a dot

PROBLEM #1

Calculate the currents in all branches of the circuit presented in the figure.
$\underline{E}_1 = 100V$, $\underline{E}_3 = 500V$, $R_1 = R_2 = R_3 = 50\Omega$, $X_M = 50\Omega$, $X_2 = X_3 = 100\Omega$.



Method 1 (Elimination of coupling)

$$\begin{cases} \underline{I}_I (R_1 + R_2 + \cancel{jX_M} + jX_2 - \cancel{jX_M}) - \underline{I}_{II} (R_2 + jX_2 - jX_M) = \underline{E}_1 \\ -\underline{I}_I (R_2 + jX_2 - jX_M) + \underline{I}_{II} (R_2 + R_3 + jX_2 - jX_M + jX_3 - jX_M) = -\underline{E}_3 \end{cases}$$

$$\underline{W} = \begin{vmatrix} 2+j2 & -1-j1 \\ -1-j1 & 2+j2 \end{vmatrix} = j6$$

$$\begin{cases} \underline{I}_I (50+50+j100) - \underline{I}_{II} (50+j100-j50) = 100 \\ -\underline{I}_I (50+j100-j50) + \underline{I}_{II} (50+50+j100-\cancel{j50}+\cancel{j100}-\cancel{j50}) = -500 \end{cases}$$

$$\underline{W}_I = \begin{vmatrix} 2 & -1-j1 \\ -10 & 2+j2 \end{vmatrix} = -6-j6$$

$$\underline{I}_I (100+j100) - \underline{I}_{II} (50+j50) = 100 \quad | : 50$$
$$-\underline{I}_I (50+j50) + \underline{I}_{II} (100+j100) = -500 \quad | : 50$$

$$\underline{W}_{II} = \begin{vmatrix} 2+j2 & 2 \\ -1-j1 & -10 \end{vmatrix} = -18-j18$$

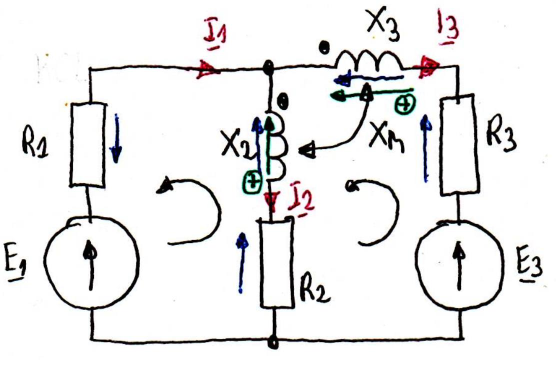
$$\begin{cases} \underline{I}_I (2+j2) - \underline{I}_{II} (1+j1) = 2 \\ -\underline{I}_I (1+j1) + \underline{I}_{II} (2+j2) = -10 \end{cases}$$

$$\underline{I}_I = \frac{\underline{W}_I}{\underline{W}} = \frac{-6-j6}{j6} = (-1+j1)\,A \qquad \underline{I}_1 = \underline{I}_I = \boxed{(-1+j1)\,A}$$

$$\underline{I}_{II} = \frac{\underline{W}_{II}}{\underline{W}} = \frac{-18-j18}{j6} = (-3+j3)\,A \qquad \underline{I}_2 = \underline{I}_I - \underline{I}_{II} = -1+j1+3-j3 = \boxed{(2-j2)\,A}$$

$$\underline{I}_3 = \underline{I}_{II} = \boxed{(-3+j3)\,A}$$

Method 2 (KCL + KVL)



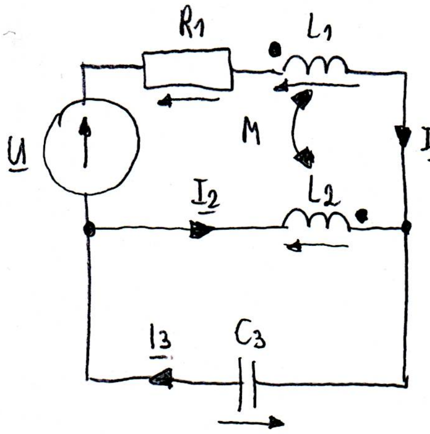
$$\begin{cases} \text{KCL:} & \underline{I}_1 - \underline{I}_2 - \underline{I}_3 = 0 \\ \text{KVL:} & R_1\underline{I}_1 + jX_2\underline{I}_2 + R_2\underline{I}_2 + jX_M\underline{I}_3 - \underline{E}_1 = 0 \\ \text{KVL:} & -R_2\underline{I}_2 - jX_2\underline{I}_2 + jX_3\underline{I}_3 + R_3\underline{I}_3 - jX_M\underline{I}_3 + jX_M\underline{I}_2 + \underline{E}_3 = 0 \end{cases}$$

PROBLEM #2

What should be the power of the Resistor R_1 so that it is not damaged during operation in the circuit shown in the figure.

$\underline{U} = 10\,V$, $R_1 = 10\,\Omega$, $X_{L1} = 25\,\Omega$, $X_{L2} = 40\,\Omega$, $X_M = 10\,\Omega$, $X_{C3} = 20\,\Omega$.

Standard resistor power: 0.125W, 0.25W, 0.4W, 0.5W, 0.6W, 0.75W, 1W, 1.2W, 2W, 3W, 5W, 7W, 8W, 9W.



$$\begin{cases} \underline{I}_1 + \underline{I}_2 = \underline{I}_3 & \leftarrow \text{KCL} \\ \underline{U} = R_1 \underline{I}_1 + jX_{L1}\underline{I}_1 \boxed{-jX_M \underline{I}_2}^{(1)} - jX_{L2}\underline{I}_2 - \left(\boxed{-jX_M \underline{I}_1}^{(1)}\right) & \leftarrow \text{KVL} \\ 0 = jX_{L2}\underline{I}_2 \underbrace{\boxed{-jX_M \underline{I}_1}}_{(1)} + (-jX_{C3}\underline{I}_3) & \leftarrow \text{KVL} \end{cases}$$

(1) $-jX_M$ when one current enters a dot and the other current leaves a dot

$+jX_M$ when both currents enter the dot (or both leave the dots)

$$10\underline{I}_1 + j25\underline{I}_1 - j10\underline{I}_2 - j40\underline{I}_2 + j10\underline{I}_1 = 10$$
$$(10+j35)\underline{I}_1 - j50\underline{I}_2 = 10$$

$$j40\underline{I}_2 - j10\underline{I}_1 - j20\underline{I}_3 = 0$$
$$\uparrow\; \underline{I}_3 = \underline{I}_1 + \underline{I}_2$$

$$j40\underline{I}_2 - j10\underline{I}_1 - j20\underline{I}_1 - j20\underline{I}_2 = 0$$
$$-j30\underline{I}_1 + j20\underline{I}_2 = 0$$
$$j20\underline{I}_2 = j30\underline{I}_1 \quad /:j20$$
$$\underline{I}_2 = \frac{3}{2}\underline{I}_1$$

$$(10+j35)\underline{I}_1 - j50\cdot\frac{3}{2}\underline{I}_1 = 10$$
$$10+j35\underline{I}_1 - j75\underline{I}_1 = 10$$
$$(10-j40)\underline{I}_1 = 10$$
$$\underline{I}_1 = \frac{10}{10-j40} = (0.0588 + j0.2353)\,A$$

$$\underline{U}_1 = R_1\cdot\underline{I}_1 = 10(0.0588 + j0.2353) = (0.588 + j2.353)\,V$$

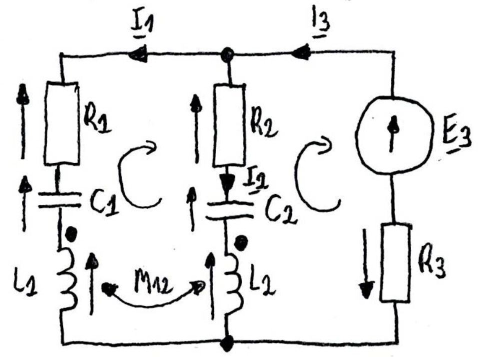
$$\underline{S}_1 = \underline{U}_1\cdot\underline{I}_1^* = (0.588+j2.353)(0.0588-j0.2353) = 0.5878\ VA$$

$$P_1 = \text{Re}\{\underline{S}_1\} = \boxed{0.5878\ W}$$

The power of resistor R_1 should be $\boxed{0.6\,W}$.

PROBLEM #3 (this problem was not solved during the class)

Write equations according to KCL and KVL for the circuit presented in the figure.



KCL → $\underline{I}_1 + \underline{I}_2 = \underline{I}_3$

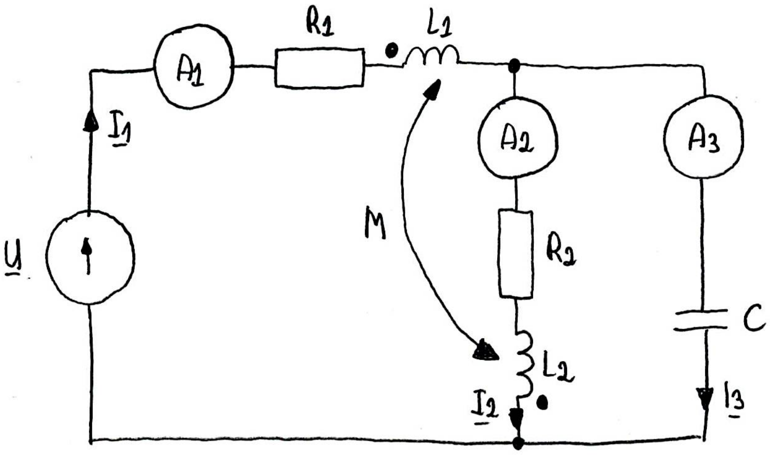
KVL → $R_1 \cdot \underline{I}_1 - jX_{C1} \cdot \underline{I}_1 + jX_{L1} \cdot \underline{I}_1 - R_2 \cdot \underline{I}_2 + jX_{C2} \cdot \underline{I}_2 - jX_{L2} \cdot \underline{I}_2 + jX_{M12} \cdot \underline{I}_2 - jX_{M12} \cdot \underline{I}_1 = 0$

KVL → $R_2 \cdot \underline{I}_2 - jX_{C2} \cdot \underline{I}_2 + jX_{L2} \cdot \underline{I}_2 + R_3 \cdot \underline{I}_3 + jX_{M12} \cdot \underline{I}_1 - \underline{E}_3 = 0$

PROBLEM #4 (this problem was not solved during the class)

Calculate meter readings in the circuit shown in the figure.

$\underline{U} = 100\,V$, $R_1 = R_2 = 15\,\Omega$, $X_{L1} = X_{L2} = 30\,\Omega$, $X_M = X_C = 10\,\Omega$.



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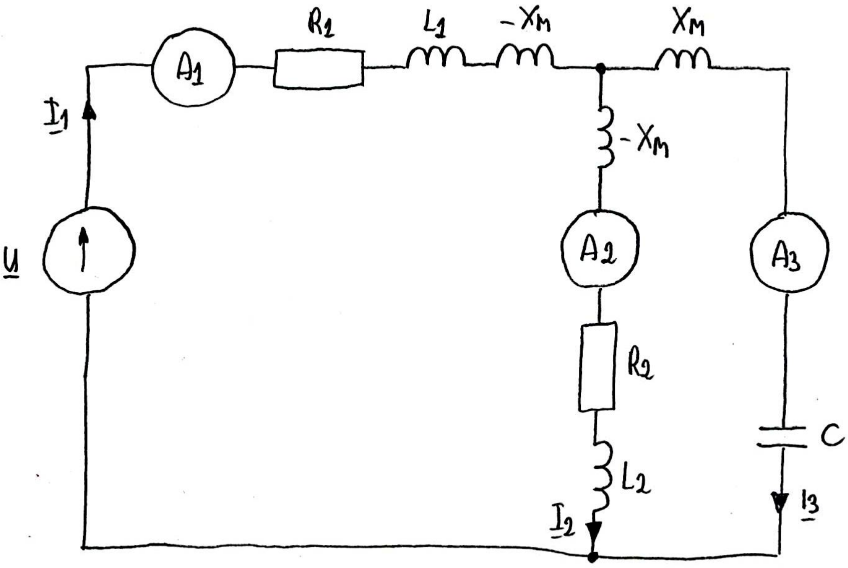

$\underline{Z}_1 = R_1 + jX_{L1} - jX_M = 15 + j30 - j10 = (15 + j20)\,\Omega$

$\underline{Z}_2 = R_2 + jX_{L2} - jX_M = 15 + j30 - j10 = (15 + j20)\,\Omega$

$\underline{Z}_3 = jX_M - jX_C = j10 - j10 = 0\,\Omega$

because $\underline{Z}_3 = 0\,\Omega$ then $\boxed{I_{A2} = 0\,A}$ and $\underline{I}_1 = \underline{I}_3$, $I_{A1} = I_{A3}$ $\quad \underline{Z}_{eq} = \underline{Z}_1 = (15 + j20)\,\Omega$

$\underline{I}_1 = \frac{\underline{U}}{\underline{Z}_{eq}} = \frac{100}{15 + j20} = (2.4 - j3.2)\,A = 4e^{-j53.1^\circ}\,A \quad \Rightarrow I_{A1} = I_{A3} = |\underline{I}_1| = \boxed{4\,A}$