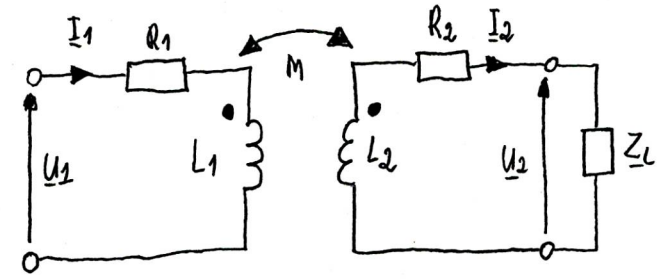
ELECTRICAL CIRCUITS 2 - CLASS NO. 4 (25.03.2025)

AIR-CORE TRANSFORMER

* circuit diagram



the primary side (the input side)

the secondary side (the output side)

$\underline{U}_1$ - primary voltage
$\underline{I}_1$ - primary current
$\underline{U}_2$ - secondary voltage
$\underline{I}_2$ - secondary current
$\underline{Z}_L$ - load impedance

* equations

$$\begin{cases} R_1\underline{I}_1 + j\omega L_1\underline{I}_1 - j\omega M\underline{I}_2 = \underline{U}_1 \\ R_2\underline{I}_2 + j\omega L_2\underline{I}_2 - j\omega M\underline{I}_1 + \underline{Z}_L\underline{I}_2 = 0 \end{cases}$$

* equations (open-circuited output), $Z_L \to \infty$, $\underline{I}_2 = 0$

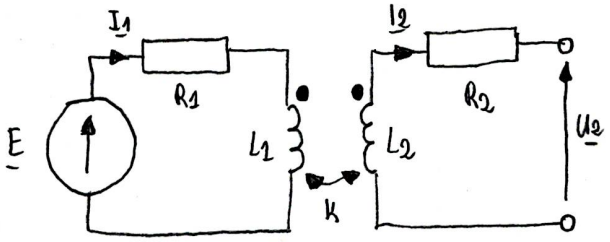
$$\begin{cases} R_1\underline{I}_1 + j\omega L_1\underline{I}_1 = \underline{U}_1 \\ -j\omega M\underline{I}_1 = \underline{U}_2 \end{cases}$$

* equations (short-circuited output), $\underline{Z}_L = 0$, $\underline{U}_2 = 0$

$$\begin{cases} R_1\underline{I}_1 + j\omega L_1\underline{I}_1 - j\omega M\underline{I}_2 = \underline{U}_1 \\ R_2\underline{I}_2 + j\omega L_2\underline{I}_2 - j\omega M\underline{I}_1 = 0 \end{cases}$$

PROBLEM #1

The air transformer was connected to the voltage source E. Calculate the readings of:
a) ammeter, b) voltmeter, connected to the terminals of the secondary winding.
$\underline{E} = 200V$, $k = 0.8$, $R_1 = R_2 = 20\,\Omega$, $X_{L1} = X_{L2} = 40\,\Omega$.



$M = k\sqrt{L_1 \cdot L_2}$ $\quad X_M = k\sqrt{X_{L1} \cdot X_{L2}} = 0.8\sqrt{40^2} = 0.8 \cdot 40 = 32\,\Omega$

a) ammeter - short circuit in the secondary winding

$$\begin{cases} R_1\underline{I}_1 + jX_{L1}\underline{I}_1 - jX_M\underline{I}_2 = \underline{E} \\ -jX_M\underline{I}_1 + R_2\underline{I}_2 + jX_{L2}\underline{I}_2 = 0 \end{cases} \quad W = \begin{vmatrix} 20+j40 & -j32 \\ -j32 & 20+j40 \end{vmatrix} = -176+j1600$$

$$\begin{cases} 20\underline{I}_1 + j40\underline{I}_1 - j32\underline{I}_2 = 200 \\ -j32\underline{I}_1 + 20\underline{I}_2 + j40\underline{I}_2 = 0 \end{cases} \quad W_I = \begin{vmatrix} 200 & -j32 \\ 0 & 20+j40 \end{vmatrix} = 4000+j8000$$

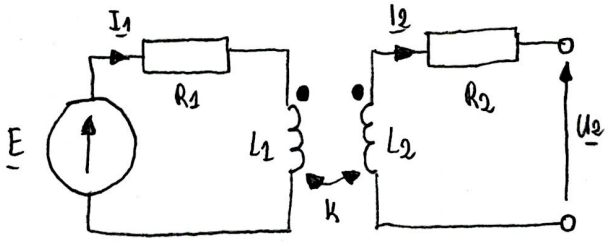
$$\begin{cases} (20+j40)\underline{I}_1 - j32\underline{I}_2 = 200 \\ -j32\underline{I}_1 + (20+j40)\underline{I}_2 = 0 \end{cases} \quad W_{II} = \begin{vmatrix} 20+j40 & 200 \\ -j32 & 0 \end{vmatrix} = 0+j6400$$

$$\underline{I}_1 = \frac{W_I}{W} = \frac{4000+j8000}{-176+j1600} = (4.6685 - j30.135)A$$

$$\underline{I}_2 = \frac{W_{II}}{W} = \frac{j6400}{-176+j1600} = (3.9522 - j0.4347)A$$

$$\boxed{|\underline{I}_2| = 3.976\,A}$$

b) voltmeter - open circuit in the secondary windings, $\underline{I}_2 = 0A$



$$R_1\underline{I}_1 + jX_{L1}\underline{I}_1 = \underline{E}$$

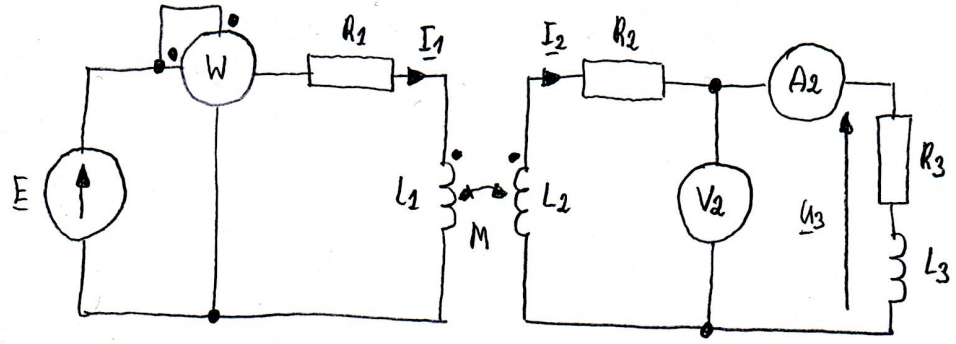
$$(R_1 + jX_{L1})\underline{I}_1 = E \rightarrow \underline{I}_1 = \frac{\underline{E}}{R_1 + jX_{L1}} = \frac{200}{20+j40} = (2-4j)A$$

$$\underline{U}_2 = -jX_M\underline{I}_1 = j32(2-4j) = (128+j64)V$$

$$\boxed{|\underline{U}_2| = 143.1084\,V}$$

PROBLEM #2

For the circuit below determine the results of measurements.
$E = 230V$, $R_1 = X_{L2} = X_M = 10\Omega$, $R_2 = 5\Omega$, $R_3 = X_{L1} = X_{L3} = 20\Omega$.



$$\begin{cases} R_1\underline{I}_1 + jX_{L1}\underline{I}_1 - jX_M\underline{I}_2 = \underline{E} \\ R_2\underline{I}_2 + jX_{L2}\underline{I}_2 - jX_M\underline{I}_1 + R_3\underline{I}_2 + jX_{L3}\underline{I}_2 = 0 \end{cases}$$

$$\begin{cases} 10\underline{I}_1 + j20\underline{I}_1 - j10\underline{I}_2 = 230 \\ 5\underline{I}_2 + j10\underline{I}_2 - j10\underline{I}_1 + 20\underline{I}_2 + j20\underline{I}_2 = 0 \end{cases}$$

$$\begin{cases} (10+j20)\underline{I}_1 + (-j10)\underline{I}_2 = 230 \\ (-j10)\underline{I}_1 + (25+j30)\underline{I}_2 = 0 \end{cases}$$

$$W = \begin{vmatrix} 10+j20 & -j10 \\ -j10 & 25+j30 \end{vmatrix} = -250 + j\,800$$

$$W_1 = \begin{vmatrix} 230 & -j10 \\ 0 & 25+j30 \end{vmatrix} = 5750 + j\,6800$$

$$W_2 = \begin{vmatrix} 10+j20 & 230 \\ -j10 & 0 \end{vmatrix} = j\,2300$$

$$\underline{I}_1 = \frac{W_1}{W} = \frac{5750+j\,6800}{-250+j\,800} = (5.81 - j\,8.00)A$$

$$\underline{I}_2 = \frac{W_2}{W} = \frac{j\,2300}{-250+j\,800} = (2.62 - j\,0.82)\,A$$

$$P_W = \mathrm{Re}[\underline{E}\cdot\underline{I}_1^*] = \mathrm{Re}[230\cdot(5.81+j\,8.00)] = \mathrm{Re}[1336.3 + j\,2070] = \boxed{1336.3\ W}$$

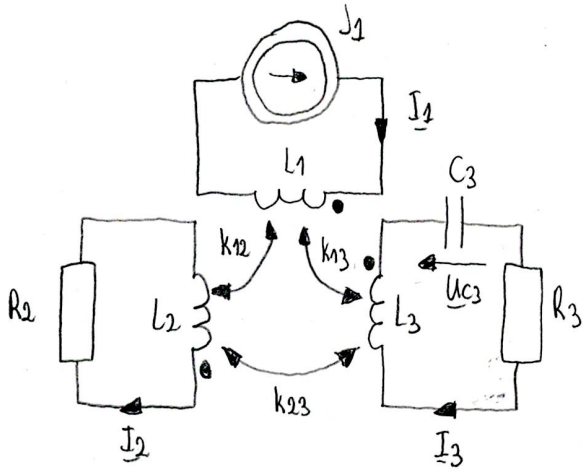
$$I_{A2} = |\underline{I}_2| = \sqrt{2.62^2 + 0.82^2} = \boxed{2.75\ A}$$

$$U_{V2} = |\underline{U}_3| = |(R_3 + jX_{L3})\cdot\underline{I}_2| = |(20+j\,20)\cdot(2.62 - j\,0.82)| = |68.8 + j\,36| = \boxed{77.65\ V}$$

PROBLEM #3

The maximum voltage across capacitor C_3 must not exceed 75 V. Check whether this condition is met in the circuit shown in the diagram.

$X_{L1}=80\Omega$, $R_2=10\Omega$, $X_{L2}=20\Omega$, $R_3=10\Omega$, $X_{L3}=20\Omega$, $X_{C3}=10\Omega$, $k_{12}=0.5$, $k_{13}=0.75$, $k_{23}=0.5$, $\underline{J}_1=3A$



$$M=k\cdot\sqrt{L_1\cdot L_2}\ /\cdot\omega$$

$$\omega M=k\cdot\sqrt{\omega L_1\cdot\omega L_2}$$

$$X_M=k\cdot\sqrt{X_{L1}\cdot X_{L2}}$$

$$X_{M12}=k_{12}\cdot\sqrt{X_{L1}\cdot X_{L2}}=0.5\cdot\sqrt{80\cdot 20}=20\Omega$$

$$X_{M13}=k_{13}\cdot\sqrt{X_{L1}\cdot X_{L3}}=0.75\cdot\sqrt{80\cdot 20}=30\Omega$$

$$X_{M23}=k_{23}\cdot\sqrt{X_{L2}\cdot X_{L3}}=0.5\cdot\sqrt{20\cdot 20}=10\Omega$$

$$\begin{cases}\underline{I}_1=\underline{J}_1\\ R_2\underline{I}_2+jX_{L2}\underline{I}_2-\underline{I}_1\,jX_{M12}+\underline{I}_3\,jX_{M23}=0\\ R_3\underline{I}_3+jX_{L3}\underline{I}_3-jX_{C3}\underline{I}_3-\underline{I}_1\,jX_{M13}+\underline{I}_2\,jX_{M23}=0\end{cases}$$

$$\begin{cases}10\underline{I}_2+j20\underline{I}_2-j20\cdot 3+j10\cdot\underline{I}_3=0\\ 10\underline{I}_3+j20\underline{I}_3-j10\underline{I}_3-j30\cdot 3+j10\underline{I}_2=0\end{cases}$$

$$\begin{cases}(10+j20)\underline{I}_2+(j10)\underline{I}_3=j60\ /:10\\ (j10)\underline{I}_2+(10+j10)\underline{I}_3=j80\ /:10\end{cases}$$

$$\begin{cases}(1+j2)\underline{I}_2+(j1)\underline{I}_3=j6\\ (j1)\underline{I}_2+(1+j1)\underline{I}_3=j8\end{cases}$$

$$D=\begin{vmatrix}1+j2 & j1\\ j1 & 1+j1\end{vmatrix}=j3$$

$$D_2=\begin{vmatrix}j6 & j1\\ j8 & 1+j1\end{vmatrix}=3+j6$$

$$D_3=\begin{vmatrix}1+j2 & j6\\ j1 & j8\end{vmatrix}=-12+j8$$

$$\underline{I}_2=\frac{D_2}{D}=\frac{3+j6}{j3}=(2-j1)\,A$$

$$\underline{I}_3=\frac{D_3}{D}=\frac{-12+j8}{j3}=(3+4j)\,A$$

$$\underline{U}_{C3}=-jX_{C3}\cdot\underline{I}_3=-j10\cdot(3+j4)=(40-30j)\,V$$

$$|\underline{U}_{C3}|=\boxed{50\ V}<75V$$

The condition is met - the voltage across capacitor C_3 is less than 75 V.