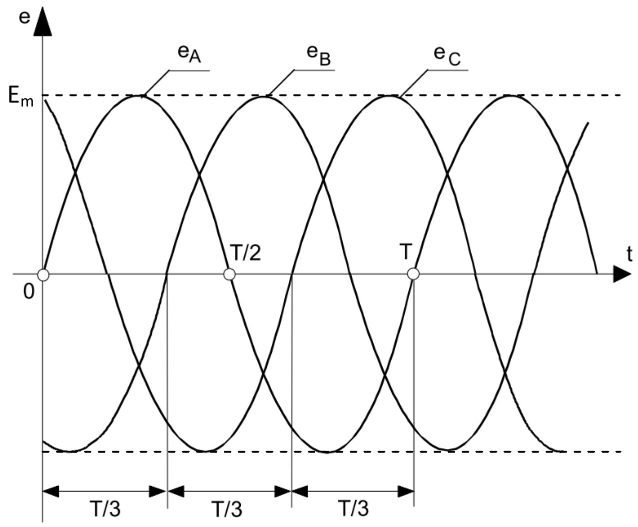
ELECTRICAL CIRCUITS 2 - CLASS 6 (08.04.2025)

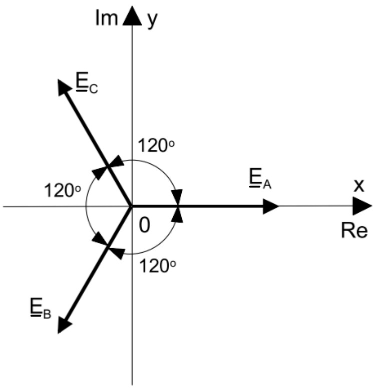
Three-phase circuits

- Generator voltages (positive phase sequence):

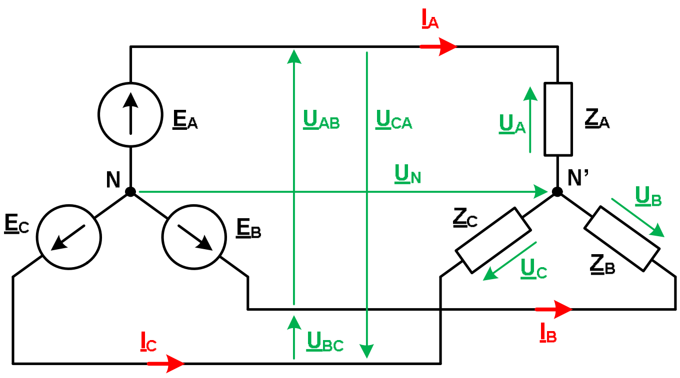
$$e_A(t) = E_m \sin(\omega t) \qquad \underline{E}_A = E e^{j0^\circ}$$
$$e_B(t) = E_m \sin(\omega t - 120^\circ) \qquad \underline{E}_B = E e^{-j120^\circ} \qquad \underline{E}_A + \underline{E}_B + \underline{E}_C = 0$$
$$e_C(t) = E_m \sin(\omega t + 120^\circ) \qquad \underline{E}_C = E e^{j120^\circ}$$

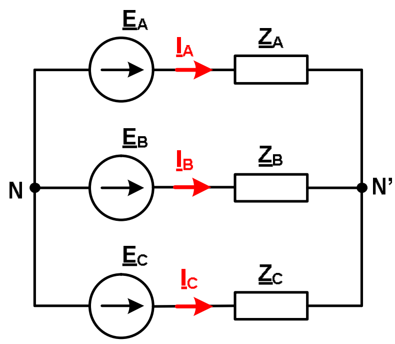
E_m - the peak (maximum) value
$\underline{E}$ - the complex value (phasor)
E - the rms (effective) value





Balanced Wye-Wye (Y-Y) connection, three-wire system





- Symbols in the circuit:

$\underline{E}_A, \underline{E}_B, \underline{E}_C$ - phase voltages of the generator
$\underline{U}_A, \underline{U}_B, \underline{U}_C$ - phase voltages of the load
$\underline{U}_{AB}, \underline{U}_{BC}, \underline{U}_{CA}$ - line voltages (line-to-line)
$\underline{U}_N$ - voltage between N and N' points

$\underline{I}_A, \underline{I}_B, \underline{I}_C$ - the line currents
$\underline{Z}_A, \underline{Z}_B, \underline{Z}_C$ - the load impedances
N - neutral point of the source
N' - neutral point of the load

- Balanced load:

$$\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = \underline{Z}$$

- $\underline{U}_N$ voltage:

$$\underline{U}_N = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} \qquad \text{where:} \qquad \underline{Y}_A = \frac{1}{\underline{Z}_A}, \quad \underline{Y}_B = \frac{1}{\underline{Z}_B}, \quad \underline{Y}_C = \frac{1}{\underline{Z}_C}$$

- Phase voltages of the load:

$\underline{U}_A = \underline{E}_A, \quad \underline{U}_B = \underline{E}_B, \quad \underline{U}_C = \underline{E}_C$ because in the system with the balanced load: $\underline{U}_N = 0$

- The line currents:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A}, \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B}, \quad \underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C}$$

the complex values (phasors): $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$

the rms (effective) values: $I_A = I_B = I_C$

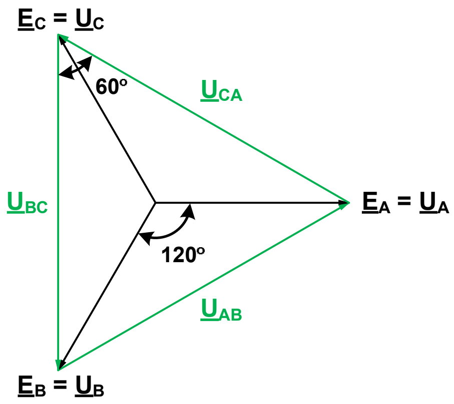
- The line voltages (line-to-line):

$$\underline{U}_{AB} = \underline{E}_A - \underline{E}_B = \sqrt{3}Ee^{j30^\circ}$$
$$\underline{U}_{BC} = \underline{E}_B - \underline{E}_C = \sqrt{3}Ee^{-j90^\circ}$$
$$\underline{U}_{CA} = \underline{E}_C - \underline{E}_A = \sqrt{3}Ee^{j150^\circ}$$

the complex values (phasors): $\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA} = 0$

the rms (effective) values: $U_{AB} = U_{BC} = U_{CA}$

- Phasor diagram of voltages:



- Active, reactive and apparent power:

$$P = U_A \cdot I_A \cdot \cos\varphi_A + U_B \cdot I_B \cdot \cos\varphi_B + U_C \cdot I_C \cdot \cos\varphi_C$$
$$Q = U_A \cdot I_A \cdot \sin\varphi_A + U_B \cdot I_B \cdot \sin\varphi_B + U_C \cdot I_C \cdot \sin\varphi_C$$
$$S = U_A \cdot I_A + U_B \cdot I_B + U_C \cdot I_C$$

Complex power:

$$\underline{S} = \underline{U}_A \cdot \underline{I}_A^* + \underline{U}_B \cdot \underline{I}_B^* + \underline{U}_C \cdot \underline{I}_C^*$$
$$\underline{S} = P + jQ$$
$$P = \mathrm{Re}\{\underline{S}\}, \quad Q = \mathrm{Im}\{\underline{S}\}, \quad S = \|\underline{S}\|$$

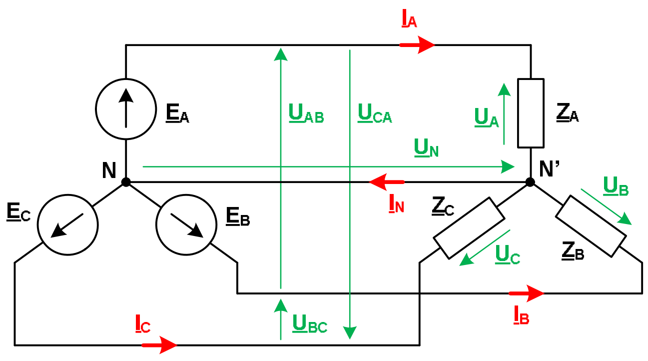
in the case of balanced load:

$$U_A = U_B = U_C = U_{ph}, \qquad I_A = I_B = I_C = I_{ph}, \qquad \cos\varphi_A = \cos\varphi_B = \cos\varphi_C = \cos\varphi_{ph}$$

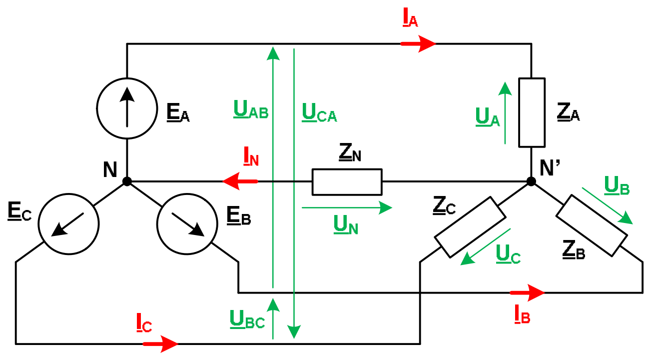
then:

$$P = 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos\varphi_{ph}, \qquad Q = 3 \cdot U_{ph} \cdot I_{ph} \cdot \sin\varphi_{ph}, \qquad S = 3 \cdot U_{ph} \cdot I_{ph}$$

Note: all of the above relationships are also valid for four-wire balanced system with impedance $\underline{Z}_N = 0$ (system on the left) or with impedance $\underline{Z}_N \neq 0$ (system on the right).



$\underline{U}_N = 0, \quad \underline{I}_N = 0$

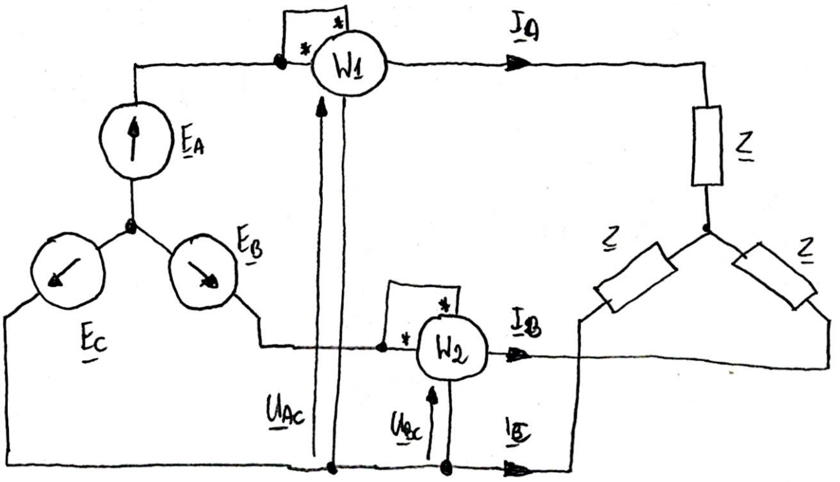


$\underline{U}_N = 0, \quad \underline{I}_N = 0$

ELECTRICAL CIRCUITS 2 - CLASS NO. 6 (08.04.2025)

PROBLEM #1

In a 3-phase balanced Y-Y system, the source voltage is $E_{phase} = 230\ V\ rms$. The impedance per phase is $\underline{Z} = (8+j6)\Omega$. Check whether an overcurrent circuit breaker with a rated current of 10 A is sufficient to protect this circuit. Also, find the active power of the load and the readings of the wattmeters.



$E_{ph} = 230\ V\ rms$

$\underline{E_A} = 230e^{j0°} = 230\ V$

$\underline{E_B} = 230e^{-j120°} = (-115 - j199.19)\ V$

$\underline{E_C} = 230e^{j120°} = (-115 + j199.19)\ V$

$\underline{I_A} = \dfrac{\underline{E_A}}{\underline{Z}} = \dfrac{230}{8+j6} = (18.4 - j13.8)A = 23e^{-j36.87°}\ A$

$\underline{I_B} = \dfrac{\underline{E_B}}{\underline{Z}} = \dfrac{-115 - j199.19}{8+j6} = (-21.15 - j9.03)A = 23e^{-j156.87}\ A$

$\underline{I_C} = \dfrac{\underline{E_C}}{\underline{Z}} = \dfrac{-115 + j199.19}{8+j6} = (2.75 + j22.83)A = 23e^{j83.13°}\ A$

$|\underline{I_A}| = 23\ A > 10\ A$

Method 1

$P = 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos\varphi_{ph}$

$U_{ph} = 230\ V \qquad I_{ph} = 23\ A \qquad \varphi_{ph} = \varphi_u - \varphi_i = 0° - (-36.87°) = 36.87°$

$P_{load} = 3 \cdot 230 \cdot 23 \cdot \cos(36.87°) = \boxed{12695.99\ W}$

Method 2

$\underline{S} = \underline{E_A} \cdot \underline{I_A}^* + \underline{E_B} \cdot \underline{I_B}^* + \underline{E_C} \cdot \underline{I_C}^* = 230 \cdot (18.4 + j13.8) + (-115 - j199.19) \cdot (-21.15 + j9.03) + (-115 + j199.19) \cdot (2.75 - j22.83)$

$= 4232 + j3174 + 4232 + j3174 + 4232 + j3174 = \underbrace{12696}_{P_{load}} + j9522\ VA$

$P_{load} = \boxed{12696\ W}$

Method 3

$\underline{U_{AC}} = \underline{E_A} - \underline{E_C} = 230 - (-115 + j199.19) = (345 - j199.19)\ V$

$\underline{U_{BC}} = \underline{E_B} - \underline{E_C} = -115 - j199.19 - (-115 + j199.19) = -j398.37\ V$

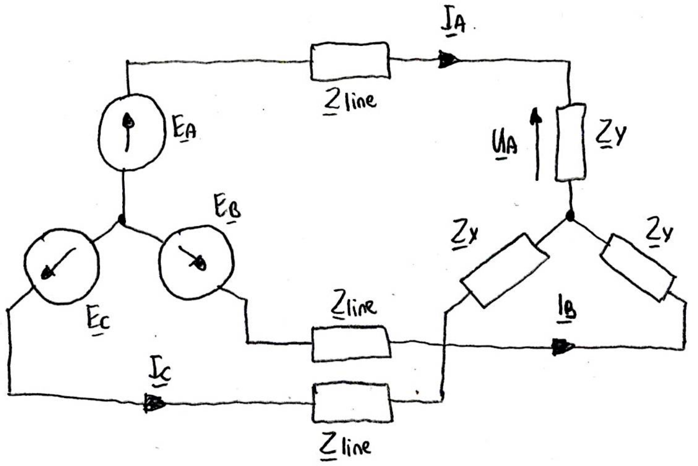
$\underline{S_1} = \underline{U_{AC}} \cdot \underline{I_A}^* = (345 - j199.19) \cdot (18.4 + j13.8) = (\underbrace{9096.76}_{P_{W1}} + j1095.98)VA \Rightarrow P_{W1} = 9096.76\ W$

$\underline{S_2} = \underline{U_{BC}} \cdot \underline{I_B}^* = (-j398.37) \cdot (-21.15 + j9.03) = (\underbrace{3599.23}_{P_{W2}} + j8426.02)VA \Rightarrow P_{W2} = 3599.23\ W$

$P_{load} = P_{W1} + P_{W2} \qquad P_{W1} + P_{W2} = 9096.76 + 3599.23 = \boxed{12695.99\ W}$

PROBLEM #2

In a 3-phase balanced Y-Y system, the source voltage is $E_{phase} = 230$ V rms. The impedance per phase is $\underline{Z}_Y = (9+9j)\Omega$ and the line impedance per phase is $(0.5+j0.4)\Omega$. What should be the rated current of the overcurrent circuit breakers protecting the circuit? Standard rated currents are: 6 A, 10 A, 16 A, 20 A, 25 A, 32 A, 40 A, 50 A, 63 A, 80 A, 125 A. Calculate the active power losses in the power line. Also, calculate the percentage voltage drop across the load compared to the rated voltage.



$$\underline{E}_A = 230e^{j0°} = 230\text{ V}$$

$$\underline{E}_B = 230e^{-j120°} = (-115 - j199.19)\text{ V}$$

$$\underline{E}_C = 230e^{j120°} = (-115 + j199.19)\text{ V}$$

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_Y + \underline{Z}_{line}} = \frac{230}{9+j9+0.5+j0.4} = \frac{230}{9.5+j9.4} =$$

$$= (12.23 - j12.10)\text{A} = 17.21\,e^{-j44.7°}\text{A}$$

$$|\underline{I}_A| = |\underline{I}_B| = |\underline{I}_C| = \boxed{17.21\text{ A}}$$

The rated current of the overcurrent circuit breaker should be: $\boxed{20\text{ A}}$

The active power losses in the power line:

$$\underline{Z}_{line} = R_{line} + j\,X_{line} = (0.5+j0.4)\Omega$$

$$P_{losses} = 3 \cdot R_{line} \cdot I_{line}^2 = 3 \cdot 0.5 \cdot 17.21^2 = 3 \cdot 148.09 = \boxed{444.27\text{ W}}$$

The percentage voltage drop:

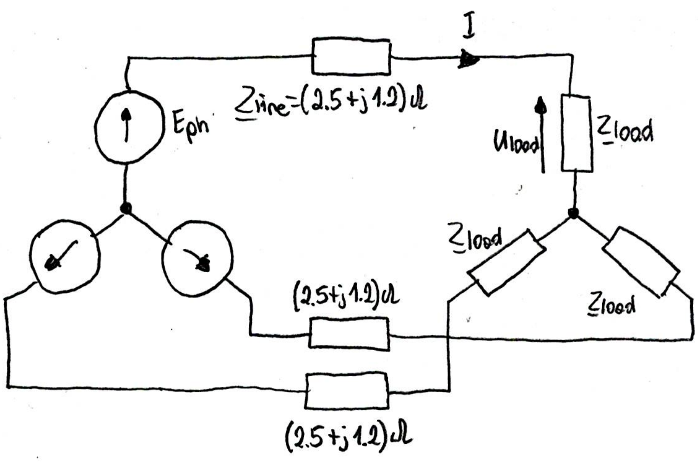
$$\underline{U}_A = \underline{I}_A \cdot \underline{Z}_Y = (12.23 - j12.10)\cdot(9+j9) = (219.04 + j1.16)\text{V} = 219.04e^{j0.3°}\text{V}$$

$$|\underline{U}_A| = 219.04\text{V} \quad |\underline{E}_A| = 230\text{ V}$$

$$\Delta U_{\%} = \frac{|\underline{E}_A| - |\underline{U}_A|}{|\underline{E}_A|} \cdot 100\% = \frac{230 - 219.04}{230} \cdot 100\% = \boxed{4.765\%}$$

PROBLEM #3

In the 3-phase balanced Y-Y system, the load voltage is $U_{load} = 400\angle{-20°}$ V rms, the line impedance is $(2.5+j1.2)\Omega$, and the source voltage is $E_{phase} = 440$ V rms. Find the load impedance, its power, and the value of supplying current.



$$\underline{I} = \frac{\underline{E}_{ph}}{\underline{Z}_{line} + \underline{Z}_{load}} \qquad \underline{I} = \frac{\underline{U}_{load}}{\underline{Z}_{load}}$$

$$\underline{E}_{ph} \cdot \underline{Z}_{load} = \underline{U}_{load}(\underline{Z}_{line} + \underline{Z}_{load})$$

$$\underline{Z}_{load}(\underline{E}_{ph} - \underline{U}_{load}) = \underline{Z}_{line}\,\underline{U}_{load}$$

$$\boxed{\underline{Z}_{load} = \underline{Z}_{line}\frac{\underline{U}_{load}}{\underline{E}_{ph} - \underline{U}_{load}}}$$

$\underline{U}_{load} = 400e^{-j20°} = (375.88 - j\,136.81)\,V$

$\underline{E}_{ph} = 440e^{j0°} = 440\,V$

$\underline{Z}_{line} = (2.5 + j\,1.2)\,\Omega$

$$\underline{Z}_{load} = (2.5+j\,1.2) \cdot \frac{375.88 - j\,136.81}{440 - 375.88 + j\,136.81} = \boxed{(3.7541 - j\,6.3081)\,\Omega}$$

$$\underline{I}_{load} = \frac{\underline{U}_{load}}{\underline{Z}_{load}} = \frac{400e^{-j20°}}{3.7541 - j\,6.3081} = (42.1947 + j\,34.4698)\,A = \boxed{54.48\,e^{j\,39.2°}\,A}$$

$$P_{load} = 3 \cdot Re\{\underline{S}_{load}\} = 3 \cdot Re\{\underline{U}_{load} \cdot \underline{I}^{*}_{load}\} = 3 \cdot Re\{(375.88 - j\,136.81)(42.1947 - j\,34.4698)\} =$$

$$= 3 \cdot Re\{11144 - j\,18729\} = 3 \cdot 11144 = \boxed{33433\ W}$$