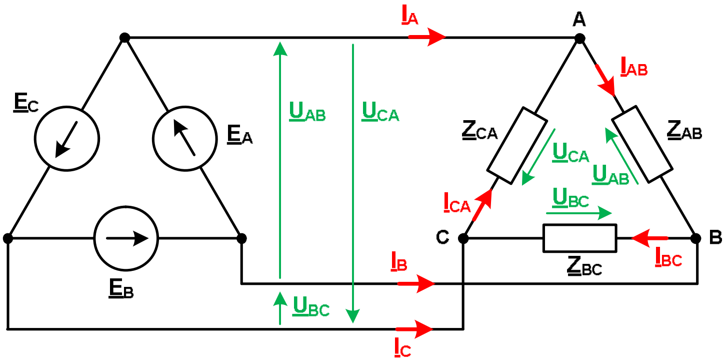
ELECTRICAL CIRCUITS 2 - CLASS 7 (15.04.2025)

Balanced Delta-Delta (Δ-Δ) connection



- Symbols in the circuit:

$\underline{E}_A, \underline{E}_B, \underline{E}_C$ - phase voltages of the generator

$\underline{U}_{AB}, \underline{U}_{BC}, \underline{U}_{CA}$ - phase voltages of the load

$\underline{I}_A, \underline{I}_B, \underline{I}_C$ - the line currents

$\underline{I}_{AB}, \underline{I}_{BC}, \underline{I}_{CA}$ - the phase currents

$\underline{Z}_{AB}, \underline{Z}_{BC}, \underline{Z}_{CA}$ - the load impedances

- Balanced load:

$\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = \underline{Z}$

- Phase voltages of the load are equal to phase voltages of the generator:

$\underline{U}_{AB} = \underline{E}_A, \quad \underline{U}_{BC} = \underline{E}_B, \quad \underline{U}_{CA} = \underline{E}_C$

the complex values (phasors): $\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA} = 0$

the rms (effective) values: $U_{AB} = U_{BC} = U_{CA}$

- The phase currents:

$\underline{I}_{AB} = \dfrac{\underline{U}_{AB}}{\underline{Z}_{AB}}, \quad \underline{I}_{BC} = \dfrac{\underline{U}_{BC}}{\underline{Z}_{BC}}, \quad \underline{I}_{CA} = \dfrac{\underline{U}_{CA}}{\underline{Z}_{CA}}$

the complex values (phasors): $\underline{I}_{AB} + \underline{I}_{BC} + \underline{I}_{CA} = 0$

the rms (effective) values: $I_{AB} = I_{BC} = I_{CA}$

- The line currents:

$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA}$

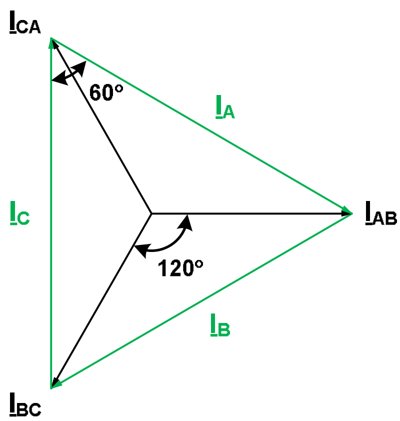
$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB}$

$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}$

the complex values (phasors): $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$

the rms (effective) values: $I_A = I_B = I_C$

- Phasor diagram of currents:



- Active, reactive and apparent power:

$$P = U_{AB} \cdot I_{AB} \cdot \cos\varphi_{AB} + U_{BC} \cdot I_{BC} \cdot \cos\varphi_{BC} + U_{CA} \cdot I_{CA} \cdot \cos\varphi_{CA}$$

$$Q = U_{AB} \cdot I_{AB} \cdot \sin\varphi_{AB} + U_{BC} \cdot I_{BC} \cdot \sin\varphi_{BC} + U_{CA} \cdot I_{CA} \cdot \sin\varphi_{CA}$$

$$S = U_{AB} \cdot I_{AB} + U_{BC} \cdot I_{BC} + U_{CA} \cdot I_{CA}$$

Complex power:

$$\underline{S} = \underline{U}_{AB} \cdot \underline{I}_{AB}^* + \underline{U}_{BC} \cdot \underline{I}_{BC}^* + \underline{U}_{CA} \cdot \underline{I}_{CA}^*$$

$$\underline{S} = P + jQ$$

$$P = \mathrm{Re}\{\underline{S}\}, \quad Q = \mathrm{Im}\{\underline{S}\}, \quad S = \|\underline{S}\|$$

in the case of balanced load:

$$U_{AB} = U_{BC} = U_{CA} = U_{ph}, \quad I_{AB} = I_{BC} = I_{CA} = I_{ph}, \quad \cos\varphi_{AB} = \cos\varphi_{BC} = \cos\varphi_{CA} = \cos\varphi_{ph}$$

then:

$$P = 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos\varphi_{ph}, \quad Q = 3 \cdot U_{ph} \cdot I_{ph} \cdot \sin\varphi_{ph}, \quad S = 3 \cdot U_{ph} \cdot I_{ph}$$

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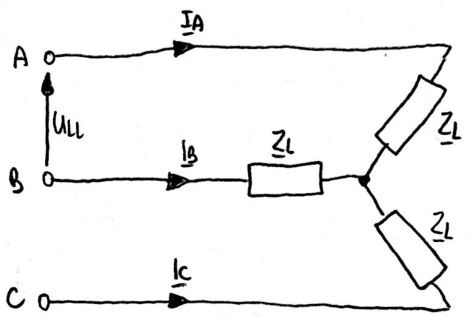
PROBLEM #1

The line-to-line voltage of a balanced 3-phase distribution line is $U_{LL} = 380$ V rms. The load impedance per phase is $\underline{Z}_L = (30+j20)\,\Omega$. Calculate the line currents and the active power of the load for the following configurations of load impedance:

a) a wye-connected system, b) a delta-connected system

a)



$$U_{ph} = \frac{U_{LL}}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 219.3931 \text{ V}$$

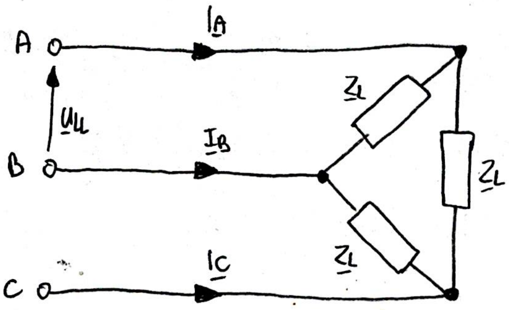
$$\underline{I}_{ph} = \frac{\underline{U}_{ph}}{\underline{Z}_L} = \frac{219.3931}{30+j20} = (5.0629 - j3.3753)\text{ A} = 6.08e^{-j33.7°}\text{ A}$$

$$I_A = I_B = I_C = I_{ph} = \boxed{6.08 \text{ A}}$$

$$\underline{S} = 3 \cdot \underline{U}_{ph} \cdot \underline{I}_{ph}^* = 3 \cdot 219.3931 \cdot (5.0629 + j3.3753) =$$
$$= (3332.3 + j2221.5) \text{ VA}$$

$$P = \text{Re}\{\underline{S}\} = \boxed{3332.3 \text{ W}}$$

b)



$$U_{ph} = U_{LL} = 380 \text{ V}$$

$$\underline{I}_{ph} = \frac{\underline{U}_{ph}}{\underline{Z}_L} = \frac{380}{30+j20} = (8.7692 - j5.8462)\text{ A} = 10.5393e^{-j33.7}$$

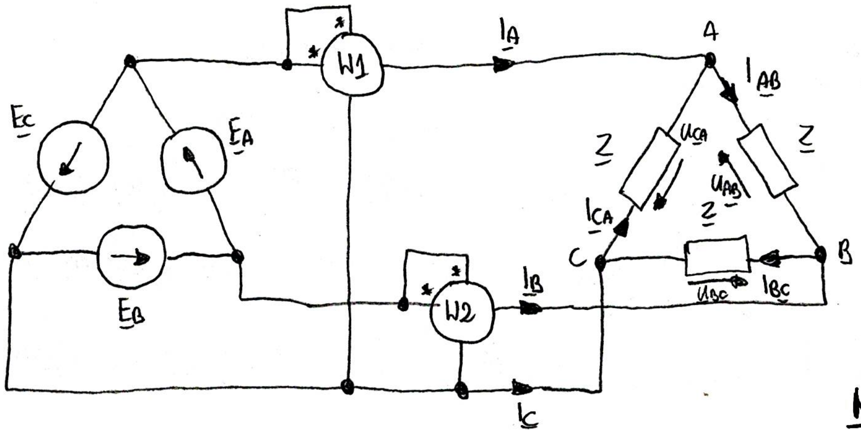
$$\underline{S} = 3\,\underline{U}_{ph} \cdot \underline{I}_{ph}^* = 3 \cdot 380 \cdot (8.7692 + j5.8462) =$$
$$= 9996.9 + j6664.6 \text{ VA}$$

$$P = \text{Re}\{\underline{S}\} = \boxed{9996.9 \text{ W}}$$

$$I_A = I_B = I_C = \sqrt{3} \cdot I_{ph} = \boxed{18.25 \text{ A}}$$

PROBLEM #2

In a 3-phase balanced Δ-Δ system the source voltage $E_{ph} = 230$ V rms. The impedance per phase is $\underline{Z} = (8+j6)\Omega$. Find the line currents, active power of the load and wattmeters readings.



$\underline{E}_A = 230e^{j0°}$ V $= 230$ V

$\underline{E}_B = 230e^{-j120°}$ V $= -115 - j199.19$ V

$\underline{E}_C = 230e^{j120°}$ V $= -115 + j199.19$ V

$\underline{U}_{AB} = \underline{E}_A$

$\underline{U}_{BC} = \underline{E}_B$

$\underline{U}_{CA} = \underline{E}_C$

Note: for I_B current, the calculated angle is positive. After changing the angle to negative (173.13° - 360° = -186.87°), we can see that the angle between the phase and line current is 30°.

$$\underline{I}_{AB} = \frac{\underline{E}_A}{\underline{Z}} = \frac{230}{8+j6} = (18.4 - j13.8)\,A = 23e^{-j36.87°}\,A$$

$$\underline{I}_{BC} = \frac{\underline{E}_B}{\underline{Z}} = \frac{-115 - j199.19}{8+j6} = (-21.15 - j9.03)\,A = 23e^{-j156.87°}\,A$$

$$\underline{I}_{CA} = \frac{\underline{E}_C}{\underline{Z}} = \frac{-115 + j199.19}{8+j6} = (2.75 + j22.83)\,A = 23e^{j83.13}\,A$$

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA} = 18.4 - j13.8 - 2.75 - j22.83 = (15.65 - j36.63)A = 39.84e^{-j66.87°}A$$

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB} = -21.15 - j9.03 - 18.4 + j13.8 = (-39.55 + j4.77)A = 39.84e^{j173.13°}A = 39.84e^{-j186.89°}A$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC} = 2.75 + j22.83 + 21.15 + j9.03 = (23.9 + j31.87)A = 39.84e^{j53.13°}A = 39.84e^{-j306.87°}A$$

$$\underline{S}_1 = (-\underline{E}_C)\cdot \underline{I}_A^* = (115 - j199.19)\cdot(15.65 + j36.63) = (\underbrace{9096.76}_{P_{W1}} + j1095.88)\,VA \Rightarrow P_{W1} = 9096.76\ W$$

$$\underline{S}_2 = \underline{E}_B \cdot \underline{I}_B^* = (-115 - j199.19)(-39.55 - j4.77) = (\underbrace{3589.23}_{P_{W2}} + j8426.02)\,VA \rightarrow P_{W2} = 3589.23\ W$$

$$\underline{S} = \underline{E}_A\cdot\underline{I}_{AB}^* + \underline{E}_B\cdot\underline{I}_{BC}^* + \underline{E}_C\cdot\underline{I}_{CA}^* = 230\cdot(18.4 - j13.8) + (-115 - j199.19)(-21.15 - j9.03) + (-115 + j199.19)(2.75 + j22.83)$$

$$= 4232 + j3174 + 4232 + j3174 + 4232 + j3174 = (\underbrace{12696}_{P_{load}} + j9522)\,VA$$

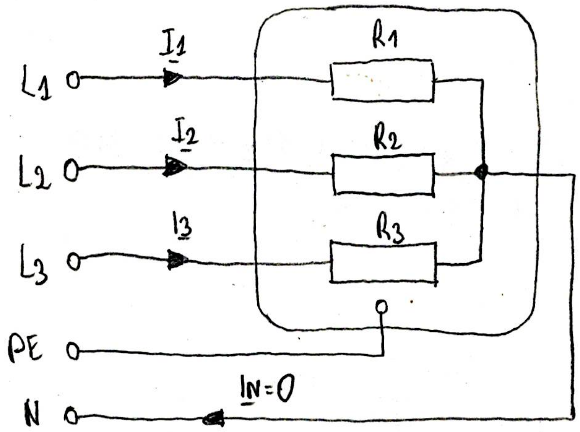
$P_{load} = P_{W1} + P_{W2}$

$P_{load} = \boxed{12696\ W}$ $\quad P_{W1} + P_{W2} = 9096.76 + 3589.23 = \boxed{12685.99\ W}$

PROBLEM #3

The three-phase electric heater consists of three heating coils Y-connected. The nominal power of the heater is $P_n = 3$ kW, and the nominal voltage $U_n = 230$ V rms. The heater has been damaged. After its repair the length of the first coil decreased by 5% and the length of the second coil by 10%.

a) calculate the line currents before repairing the heater.



$$P = \frac{P_n}{3} = \frac{3000}{3} = 1000\ \text{W}$$

$$R_1 = R_2 = R_3 = R \qquad R = \frac{U_n^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega$$

$$\underline{E}_1 = 230e^{j0°} = 230\ \text{V}$$

$$\underline{E}_2 = 230e^{-j120°} = (-115 - j\,199.19)\ \text{V}$$

$$\underline{E}_3 = 230e^{+j120°} = (-115 + j\,199.19)\ \text{V}$$

$$\underline{I}_1 = \frac{\underline{E}_1}{R} = \frac{230}{52.9} = 4.3478 = 4.3478e^{j0°}\ \text{A}$$

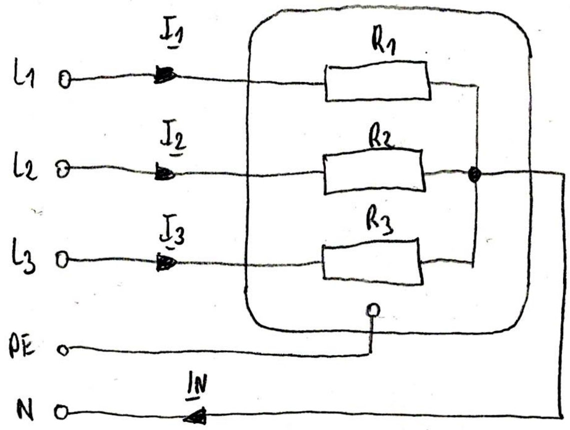
$$\underline{I}_2 = \frac{\underline{E}_2}{R} = \frac{-115 - j\,199.19}{52.9} = (-2.1739 - j\,3.7653) = 4.3478e^{-j120°}\ \text{A}$$

$$\underline{I}_3 = \frac{\underline{E}_3}{R} = \frac{-115 + j\,199.19}{52.9} = (-2.1739 + j\,3.7653) = 4.3478e^{+j120°}\ \text{A}$$

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$I_1 = I_2 = I_3$$

b) calculate line currents, the current in the neutral line and the power of the repaired heater



$$R_1 = 0.95R = 0.95 \cdot 52.9 = 50.255\ \Omega$$

$$R_2 = 0.9R = 0.9 \cdot 52.9 = 47.61\ \Omega$$

$$R_3 = R = 52.9\ \Omega$$

$$\underline{I}_1 = \frac{\underline{E}_1}{R_1} = \frac{230}{50.255} = 4.5767 = 4.5767e^{j0°}\ \text{A}$$

$$\underline{I}_2 = \frac{\underline{E}_2}{R_2} = \frac{(-115 - j\,199.19)}{47.61} = (-2.4155 - j\,4.1837) = 4.8309e^{-j120°}\ \text{A}$$

$$\underline{I}_3 = \frac{\underline{E}_3}{R_3} = \frac{(-115 + j\,199.19)}{52.9} = (-2.1739 + j\,3.7653) = 4.3478e^{+j120°}\ \text{A}$$

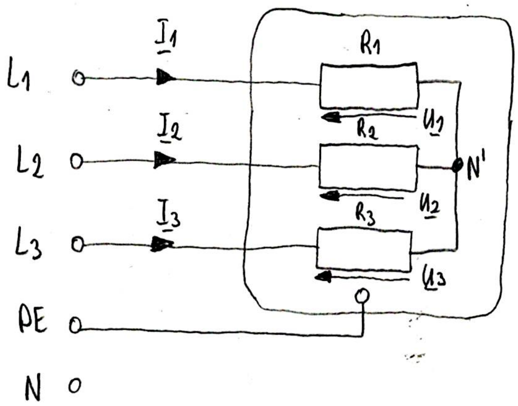
$$\underline{I}_N = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 4.5767 - 2.4155 - j\,4.1837 - 2.1739 + j\,3.7653 = (-0.0127 - j\,0.4184) = 0.4186e^{-j91.74°}\ \text{A}$$

$$P_n = \frac{U_n^2}{R_1} + \frac{U_n^2}{R_2} + \frac{U_n^2}{R_3} = \frac{230^2}{50.255} + \frac{230^2}{47.61} + \frac{230^2}{52.9} = 3163.7\ \text{W}$$

$$P_n = \text{Re}\{\underline{S}\} = \text{Re}\{\underline{E}_1 \cdot \underline{I}_1^* + \underline{E}_2 \cdot \underline{I}_2^* + \underline{E}_3 \cdot \underline{I}_3^*\} = \text{Re}\{230 \cdot (4.5767) + (-115 - j\,199.19)(-2.4155 + j\,4.1837) +$$

$$+ (-115 + j\,199.19)(-2.1739 - j\,3.7653)\} = 3163.7\ \text{W}$$

c) calculate line currents and the power of the repaired heater, when the neutral line is not connected



$$\underline{Y}_1 = \frac{1}{\underline{Z}_1} = \frac{1}{R_1} = \frac{1}{50.255} = 0.0199\,\Omega^{-1}$$

$$\underline{Y}_2 = \frac{1}{\underline{Z}_2} = \frac{1}{R_2} = \frac{1}{47.61} = 0.021\,\Omega^{-1}$$

$$\underline{Y}_3 = \frac{1}{\underline{Z}_3} = \frac{1}{R_3} = \frac{1}{52.9} = 0.0189\,\Omega^{-1}$$

$$\underline{U}_{NN'} = \frac{\underline{E}_1\cdot\underline{Y}_1 + \underline{E}_2\cdot\underline{Y}_2 + \underline{E}_3\cdot\underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} = \frac{230\cdot 0.0199 + (-115 - j199.19)\cdot 0.021 + (-115 + j199.19)\cdot 0.0189}{0.0199 + 0.021 + 0.0189} =$$

$$= (-0.2126 - j\,6.9954)\,V$$

$$\underline{U}_1 = \underline{E}_1 - \underline{U}_{NN'} = 230 - (-0.2126 - j\,6.9954) = (230.2126 + j\,6.9954)\,V = 230.3188\,e^{j1.74°}\,V$$

$$\underline{U}_2 = \underline{E}_2 - \underline{U}_{NN'} = (-115 - j\,199.19) - (-0.2126 - j\,6.9954) = (-114.7874 - j\,192.1904)\,V = 223.86\,e^{-j120.85°}\,V$$

$$\underline{U}_3 = \underline{E}_3 - \underline{U}_{NN'} = (-115 + j\,199.19) - (-0.2126 - j\,6.9954) = (-114.7874 + j\,206.1813)\,V = 235.9807\,e^{j119.11°}\,V$$

$$\underline{I}_1 = \frac{\underline{U}_1}{R_1} = \frac{230.2126 + j\,6.9954}{50.255} = (4.5809 + j\,0.1392)\,A = 4.583\,e^{j1.74°}\,A$$

$$\underline{I}_2 = \frac{\underline{U}_2}{R_2} = \frac{-114.7874 - j\,192.1904}{47.61} = (-2.411 - j\,4.0368)\,A = 4.701\,e^{-j120.85°}\,A$$

$$\underline{I}_3 = \frac{\underline{U}_3}{R_3} = \frac{-114.7874 + j\,206.1813}{52.9} = (-2.1699 + j\,3.8976)\,A = 4.4609\,e^{j119.11°}\,A$$

$$P_n = \frac{|\underline{U}_1|^2}{R_1} + \frac{|\underline{U}_2|^2}{R_2} + \frac{|\underline{U}_3|^2}{R_3} = \frac{230.3188^2}{50.255} + \frac{223.86^2}{47.61} + \frac{235.9807^2}{52.9} = 3160.8\,W$$

$$P_n = R_1\cdot|\underline{I}_1|^2 + R_2\cdot|\underline{I}_2|^2 + R_3\cdot|\underline{I}_3|^2 = 50.255\cdot 4.583^2 + 47.61\cdot 4.701^2 + 52.9\cdot 4.4609^2 = 3160.8\,W$$

$$P_n = Re\{\underline{S}\} = Re\{\underline{U}_1\cdot\underline{I}_1^* + \underline{U}_2\cdot\underline{I}_2^* + \underline{U}_3\cdot\underline{I}_3^*\} = 3160.8\,W$$