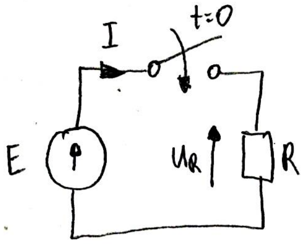
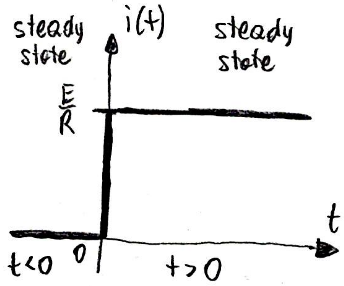
ELECTRICAL CIRCUITS 2 - CLASS NO. 10 (20.05.2025)

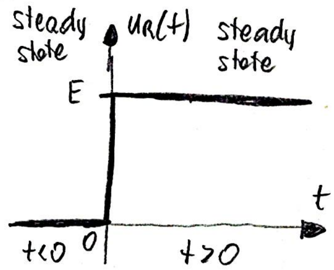
TRANSIENT AND STEADY STATE

* R CIRCUIT



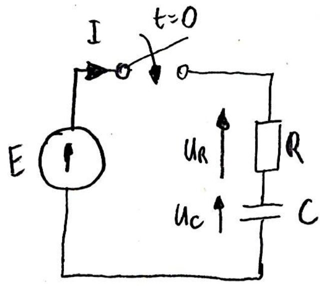
$t<0$	$t>>0$
$I=0$	$I=\frac{E}{R}$
$U_R=0$	$U_R=E$



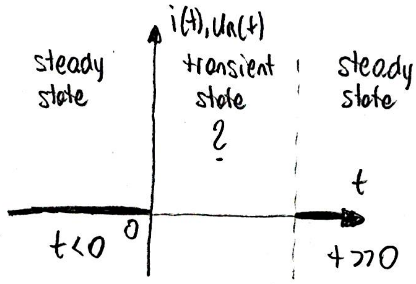


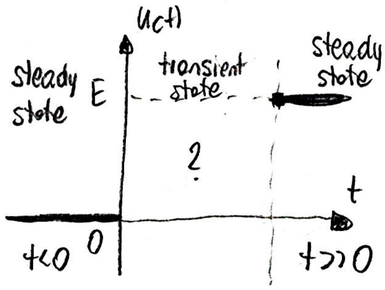
E - DC voltage source

* RC CIRCUIT

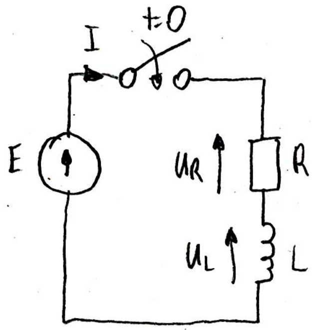


$t<0$	$t>>0$
$I=0$	$I=0$
$U_R=0$	$U_R=0$
$U_C=0$	$U_C=E$

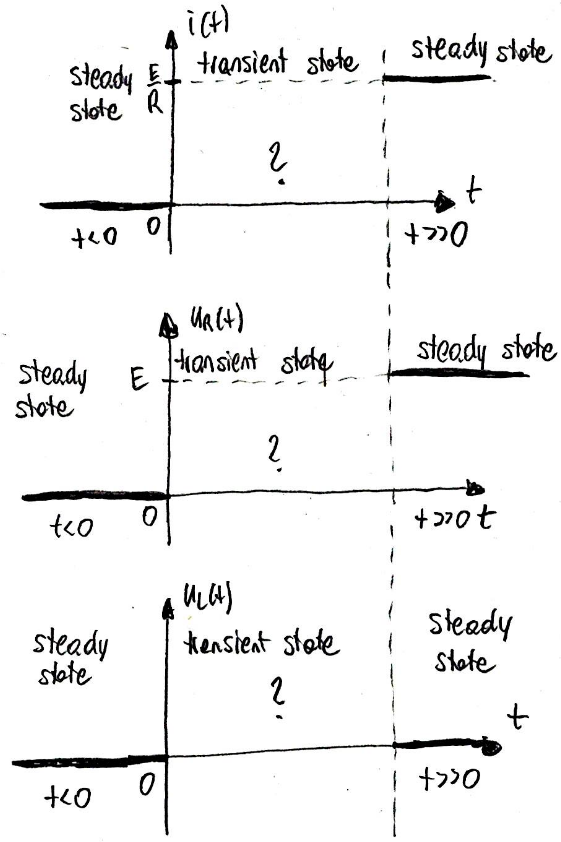




* RL CIRCUIT



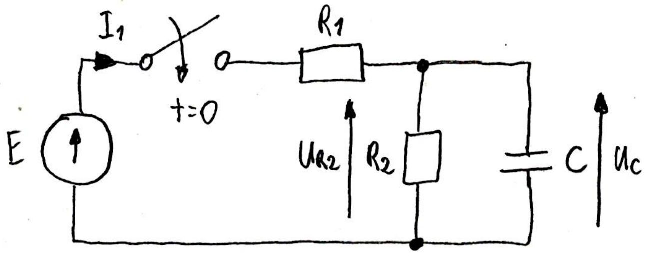
$t<0$	$t>>0$
$I=0$	$I=\frac{E}{R}$
$U_R=0$	$U_R=E$
$U_L=0$	$U_L=0$



INITIAL AND STEADY STATE VALUES

* initial conditions
  - for C → U_C at $t=0$
  - for L → I_L at $t=0$

* example 1



$t<0$

$\boxed{U_C=0}$

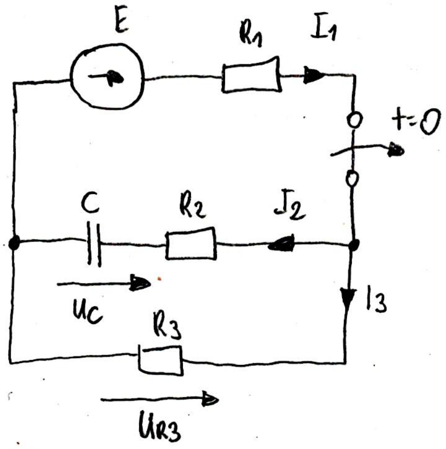
$t>>0$

$U_C=U_{R2}$

$I_1=\frac{E}{R_1+R_2}$ $\quad U_{R2}=U_C=I_1 \cdot R_2=\frac{R_2 \cdot E}{R_1+R_2}$

$$\boxed{U_C=\frac{R_2 \cdot E}{R_1+R_2}}$$

* example 2



$t<0$

$U_C=U_{R3}$

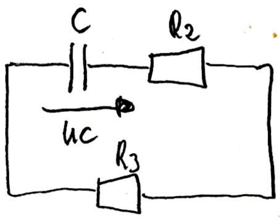
$I_2=0$, C → open circuit for DC

$I_1=I_3$

$I_1=\frac{E}{R_1+R_3}$ $\quad U_{R3}=I_1 R_3=\frac{R_3 E}{R_1+R_3}$

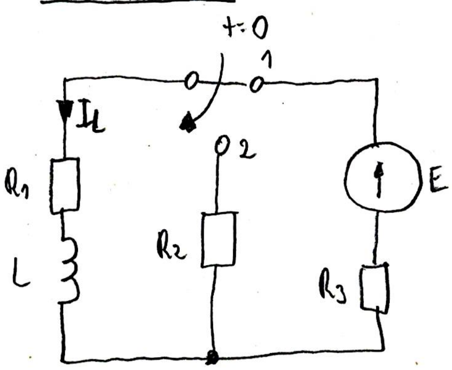
$$\boxed{U_C=\frac{R_3 \cdot E}{R_1+R_3}}$$

$t>>0$



no source → $\boxed{U_C=0}$

* example 3

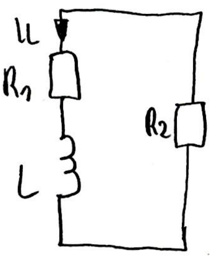


$t<0$

L → short circuit for DC

$$\boxed{I_L=\frac{E}{R_1+R_3}}$$

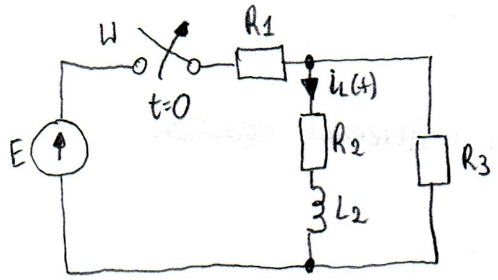
$t>>0$



no source → $\boxed{I_L=0}$

PROBLEM #1

The circuit shown in the figure has been in a steady-state. The switch was open at $t=0$. Find $i_L(t)$. $E=80V$, $R_1=120\Omega$, $R_2=50\Omega$, $R_3=200\Omega$, $L_2=0.75\,H$.

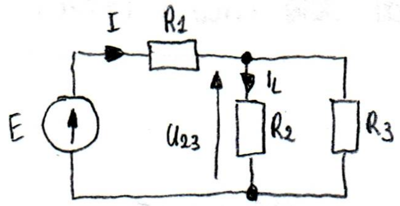


* INITIAL VALUES ($t<0$), steady-state before opening the switch

Notes:

- we consider circuit as a DC circuit
- in the case of a DC circuit, we replace inductor with a short-circuit and capacitor with an open-circuit
- we use classical methods ; Ohm's Law, KCL, KVL

our circuit before opening the switch (DC circuit), $I_L=?$



$$I=\frac{E}{R_1+\frac{R_2\cdot R_3}{R_2+R_3}}=\frac{80}{120+\frac{50\cdot 200}{50+200}}=\frac{80}{120+40}=\frac{80}{160}=0{,}5\,A$$

$$U_{23}=I\cdot\frac{R_2\cdot R_3}{R_2+R_3}=0.5\cdot\frac{50\cdot 200}{50+200}=0.5\cdot 40=20\,V$$

$$I_L=\frac{U_{23}}{R_2}=\frac{20}{50}=0.4\,A$$

* TRANSIENT ANALYSIS ($t>0$), circuit after opening the switch

Notes:

- we write differential equation with the use of KVL or KCL
- note that : $u_L=L\frac{di_L(t)}{dt}$ $\quad i_C(t)=C\frac{du_C(t)}{dt}$
- general form of differential equation:

$$a_n\frac{d^n x(t)}{dt^n}+a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}}+\ldots+a_1\frac{dx(t)}{dt}+a_0x(t)=f(t) \qquad (1)$$

where:

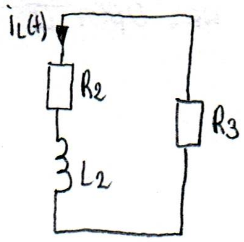
$x(t)$ - searched quantity (current / voltage)

$a_n, a_{n-1},\ldots, a_1, a_0$ - constant coefficients (R,L,C,M in the circuit)

$f(t)$ - voltage and current sources

n - the order of differential equation (the number of inductors / capacitors in the circuit)

Our circuit after opening the switch



using KVL:

$$u_{R_2}(t) + u_{L_2}(t) + u_{R_3}(t) = 0$$

$$R_2 \cdot i_L(t) + L_2 \frac{di_L(t)}{dt} + R_3 \cdot i_L(t) = 0$$

$$\boxed{L_2 \frac{di_L(t)}{dt} + (R_2+R_3)\, i_L(t) = 0}$$ - a first-order differential equation

Notes:
- the solution of equation (1) has a form

$$x(t) = x_p(t) + x_c(t) \qquad (2)$$

where:

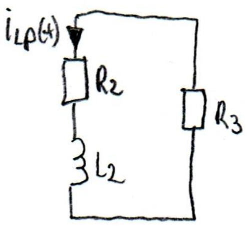
$x_p(t)$ - the particular integral solution (forced response)

$x_c(t)$ - the complementary solution (natural response)

in our case: $i_L(t) = i_{Lp}(t) + i_{Lc}(t)$

Notes:
- the particular integral solution we obtain by solving the steady state circuit $(t>>0)$

in steady state $(t>>0)$ $i_{Lp}(t) = ?$



$i_{Lp}(t) = 0$ ← because there is no voltage source in the circuit

Notes:
- we obtain the complementary solution by solving the homogeneous equation (1)

$$a_n \frac{d^n x_c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x_c(t)}{dt^{n-1}} + \dots + a_1 \frac{dx_c(t)}{dt} + a_0 x_c(t) = 0 \qquad (3)$$

- the solution of (3) has a form:

$$x_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \dots A_n e^{s_n t}$$

where:

$s_1, s_2, \dots, s_n$ - roots of characteristic equation: $a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0$

$A_1, A_2, \dots A_n$ - integration constants determined with the use of initial values

$i_{Lc}(t)$ - the complementary solution

a first-order differential equation:

$$\boxed{L_2 \frac{di_{Lc}(t)}{dt} + (R_2+R_3)\, i_{Lc}(t) = 0}$$

has a solution in the form:

$$\boxed{i_{Lc}(t) = A_1 \cdot e^{S_1 t}}$$

S_1 is determined from the characteristic equation:

$$L_2 \cdot S_1 + (R_2+R_3) = 0 \quad \Rightarrow \quad \boxed{S_1 = -\frac{R_2+R_3}{L_1} = -\frac{50+200}{0.75} = -333.33}$$

A_1 is determined with the use of initial conditions

at $t=0$: $i_L(0^-) = i_L(0^+)$ $\qquad\qquad$ $i_L(0^-) = i_L(0^+) = I_L = 0.4\,A$

$$i_L(t) = i_{Lp}(t) + i_{Lc}(t)$$

$$i_L(t) = 0 + A_1 e^{S_1 t}$$

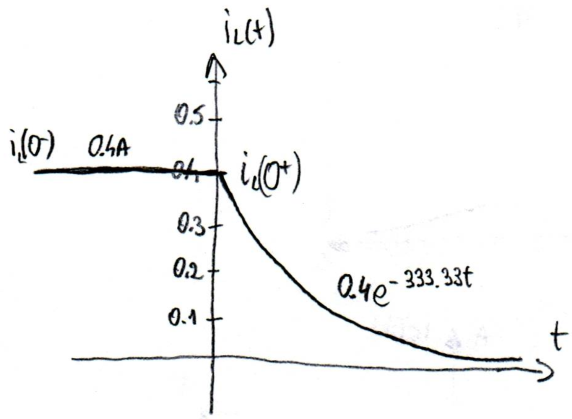
$$i_L(t) = A_1 e^{-\frac{R_2+R_3}{L_2} \cdot t}$$

$$t=0 \Rightarrow \quad 0.4 = A_1 e^{-\frac{R_2+R_3}{L_2} \cdot 0} = 1$$

$$\boxed{A_1 = 0.4}$$

finally:

$$i_L(t) = i_{Lp}(t) + i_{Lc}(t) = 0 + 0.4e^{-333.3t} = \boxed{0.4e^{-333.3t}\ A}$$



$$u_{L2}(t) = L_2 \frac{di_L(t)}{dt} = 0.75 \frac{d}{dt}\left(0.4e^{-333.3t}\right) =$$

$$= 0.75 \cdot 0.4 \cdot (-333.3)e^{-333.3t} = \boxed{-100e^{-333.3t}}$$