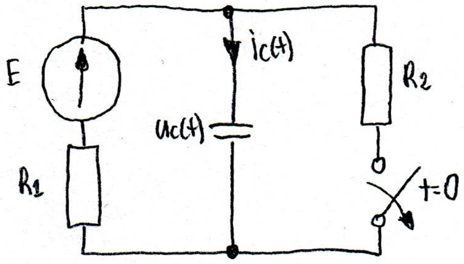
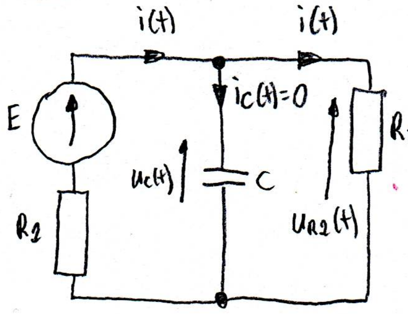
ELECTRICAL CIRCUITS 2 - CLASS NO. 11 (27.05.2025)

PROBLEM #1

The circuit shown in the figure has been in a steady-state. The switch was open at $t=0$. Calculate and plot $i_C(t)$ and $u_C(t)$ for $t<0$, $t=0$ and $t>0$. $E=100V$, $R_1=20\Omega$, $R_2=40\Omega$, $C=10mF$.



steady-state before opening the switch ($t<0$)



$i_C(t) = 0\,A$

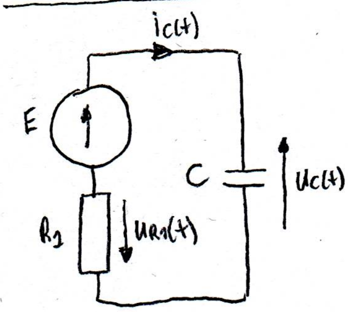
$u_C(t) = u_{R2}(t)$

$i(t) = \frac{E}{R_1+R_2}$

$u_{R2}(t) = R_2 \cdot i(t) = \frac{R_2}{R_1+R_2} \cdot E$

$u_{R2}(t) = \frac{40}{20+40} \cdot 100 = 66{,}67\,V$

circuit after opening the switch ($t>0$)



$u_{R1}(t) + u_C(t) = E$

$R_1 \cdot i_C(t) + u_C(t) = E$

$i_C(t) = C\frac{du_C(t)}{dt}$

$\boxed{R_1 C \frac{du_C(t)}{dt} + u_C(t) = E}$

$\boxed{u_C(t) = u_{Cp}(t) + u_{Cc}(t)}$

$u_{Cp}(t)$ - the particular solution ($t>>0$)

$\boxed{u_{Cp}(t) = E = 100\,V}$

$u_{Cc}(t)$ - the complementary solution

$R_1 C \frac{du_{Cc}(t)}{dt} + u_{Cc}(t) = 0 \qquad u_{Cc}(t) = A_1 e^{s_1 t}$

$R_1 C s_1 + 1 = 0 \Rightarrow \quad s_1 = -\frac{1}{R_1 C} = -\frac{1}{20 \cdot 0.01} = -\frac{1}{0.2} = -5\,s$

at $t=0$

$u_C(t) = u_{Cp}(t) + u_{Cc}(t)$

$66{,}67 = 100 + A_1 e^0 \Rightarrow \quad A_1 = 66{,}67 - 100 = -33{,}33\,V$

$\boxed{u_{Cc}(t) = -33.33 e^{-5t}\,V}$

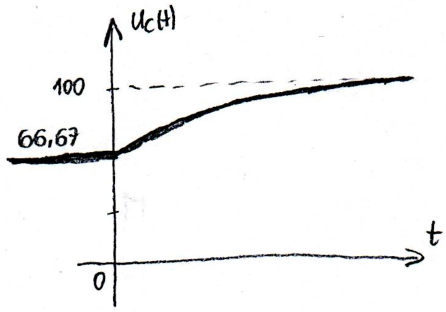
$u_C(t) = u_{Cp}(t) + u_{Cc}(t) = \boxed{100 - 33{,}33 e^{-5t}\,V}$

$i_C(t) = C\frac{du_C(t)}{dt} = 0.01 \frac{d}{dt}\left(100 - 33{,}33 e^{-5t}\right) = 0.01 \cdot 33{,}33 \cdot 5 e^{-5t} = \boxed{1.67 e^{-5t}\,A}$

$u_C(t<0) = 66.67\,V$

$u_C(t=0) = 66.67\,V$

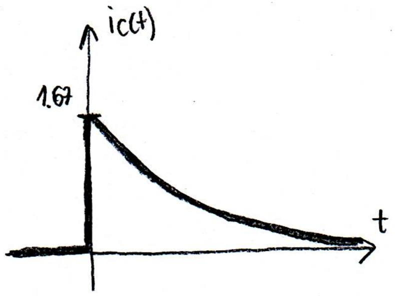
$u_C(t\to\infty) = 100\,V$



$i_C(t<0) = 0\,A$

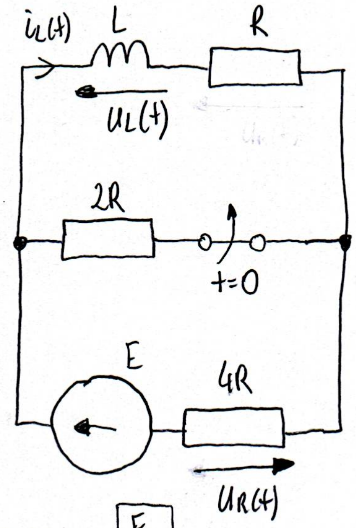
$i_C(t=0) = 1.67\,A$

$i_C(t\to\infty) = 0\,A$

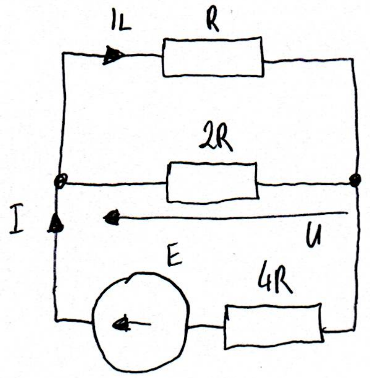


PROBLEM #2

The circuit shown in the figure has been in a steady-state. At $t=0$ the switch was open. Find and plot $i_L(t)$, $u_L(t)$, and $u_R(t)$ for $t<0$, $t=0$, and $t>0$.



steady-state before opening the switch ($t<0$)



$$R_{eq1} = \frac{R \cdot 2R}{R+2R} = \frac{2R^2}{3R} = \frac{2}{3}R$$

$$R_{eq} = \frac{2R}{3} + 4R = \frac{2R}{3} + \frac{12R}{3} = \frac{14}{3}R$$

$$I = \frac{E}{R_{eq}} = \frac{E\,3}{14R} = \frac{3E}{14R}$$

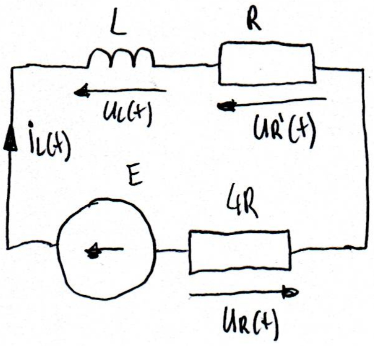
$$u = R_{eq1} \cdot I = \frac{2R}{3} \cdot \frac{3E}{14R} = \frac{E}{7}$$

$$I_L = \frac{u}{R} = \frac{E}{7R}$$

$$I_L(0^-) = \boxed{\frac{E}{7R}}$$

$$u_L(0^-) = \boxed{0} \qquad u_R(0^-) = 4R \cdot I = \frac{4R \cdot 3E}{14R} = \boxed{\frac{6}{7}E}$$

circuit after opening the switch ($t>0$)



KVL

$$u_L(t) + R \cdot i_L(t) + 4R \cdot i_L(t) = E$$

$$\boxed{L\frac{di_L(t)}{dt} + 5R \cdot i_L(t) = E}$$

$$i_L(t) = i_{Lp}(t) + i_{Lc}(t)$$

$i_{Lp}(t)$ - the particular solution ($t \gg 0$)

$$\boxed{i_{Lp}(t) = \frac{E}{5R}}$$

$i_{Lc}(t)$ - the complementary solution

$$L\frac{di_{Lc}(t)}{dt} + 5R\, i_{Lc}(t) = 0$$

$$i_{Lc}(t) = A_1 e^{S_1 t}$$

$$\boxed{i_{Lc} = -\frac{2E}{35R} e^{\frac{-5R}{L}t}}$$

S_1

$$L S_1 + 5R = 0$$

$$S_1 = -\frac{5R}{L}$$

A_1

$$i_L(0^-) = i_L(0^+)$$

$$i_L(t) = i_{Lp}(t) + i_{Lc}(t) \quad \text{for } t=0$$

$$\frac{E}{7R} = \frac{E}{5R} + A_1 \boxed{e^{S_1 t}}$$

$t=0$

$=1$

$$A_1 = \frac{E}{7R} - \frac{E}{5R} = \frac{5E-7E}{35R} = -\frac{2E}{35R}$$

$i_L(t) = i_{Lp}(t) + i_{Lc}(t)$

$$\boxed{i_L(t) = \frac{E}{5R} - \frac{2E}{35R} e^{-\frac{5R}{L}t}}$$

$$u_L(t) = L\frac{di_L(t)}{dt} = L\frac{d}{dt}\left(\frac{E}{5R} - \frac{2E}{35R} e^{-\frac{5R}{L}t}\right) = -\frac{2E\cancel{L}}{\cancel{35R}_{\,7}}\left(-\frac{\cancel{5R}^{\,1}}{\cancel{L}}\right) e^{-\frac{5R}{L}t} =$$

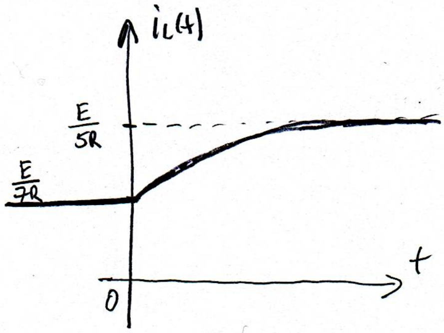
$$= \boxed{\frac{2E}{7} e^{-\frac{5R}{L}t}}$$

$$u_R(t) = 4R \cdot i_L(t) = \frac{4\cancel{R}E}{5\cancel{R}} - \frac{2\cdot 4\cancel{R}E}{35\cancel{R}} e^{-\frac{5R}{L}t} = \boxed{\frac{4E}{5} - \frac{8E}{35} e^{-\frac{5R}{L}t}}$$

$i_L(t<0) = \frac{E}{7R}$

$i_L(t=0) = \frac{E}{5R} - \frac{2E}{35R} = \frac{7E - 2E}{35R} = \frac{E}{7R}$

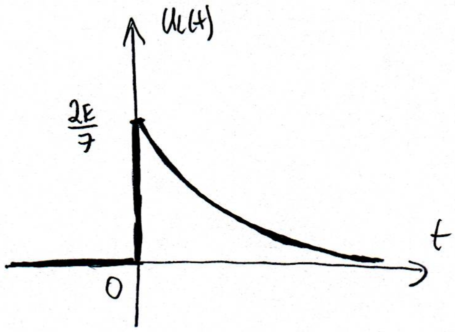
$i_L(t>0) = \frac{E}{5R}$



$u_L(t<0) = 0$

$u_L(t=0) = \frac{2E}{7}$

$u_L(t>0) = 0$

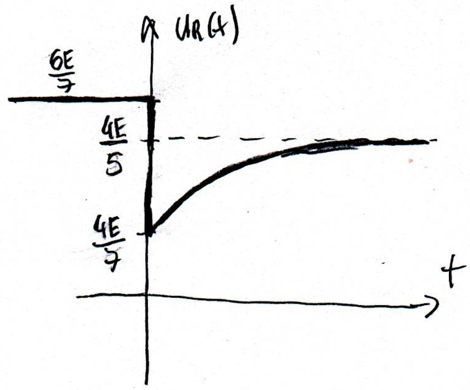


$u_R(t<0) = \frac{6E}{7}$

$u_R(t>0) = \frac{4E}{5}$

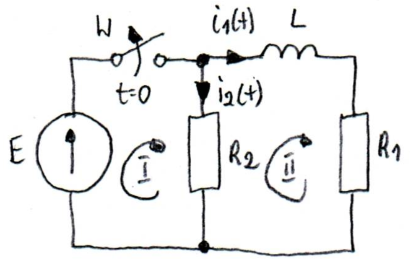
$u_R(t=0) = \frac{4E}{5} - \frac{8E}{35} = \frac{(28-8)E}{35} =$

$= \frac{20E}{35} = \frac{4E}{7}$



PROBLEM #3

The circuit shown in the figure has been in a steady-state. The switch was open at $t=0$. Plot $i_1(t)$ and $i_2(t)$ for $t<0$, $t=0$ and $t>0$. Use the classical method. $E=100V$, $L=0.1H$, $R_1=25\Omega$, $R_2=75\Omega$.



steady-state before opening switch ($t<0$)

$$\begin{cases} I_I R_2 - I_{II} R_2 = E \\ -I_I R_2 + I_{II}(R_1+R_2) = 0 \end{cases}$$

$$\begin{cases} 75 I_I - 75 I_{II} = 100 \\ -75 I_I + 100 I_{II} = 0 \end{cases} +$$

$$25 I_{II} = 100$$

$$I_{II} = 4\,A$$

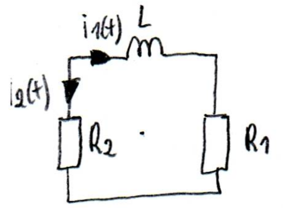
$$I_I R_2 = E + I_{II} R_2$$

$$I_I = \frac{E + I_{II} R_2}{R_2} = \frac{100 + 4 \cdot 75}{75} = \frac{400}{75} = 5.33\,A$$

$$i_1 = I_{II} = \boxed{4A}$$

$$i_2 = I_I - I_{II} = 5{,}33 - 4 = \boxed{1{,}33\,A}$$

circuit after opening the switch ($t>0$)



using KVL:

$$i_1(t) \cdot R_1 + i_1(t) \cdot R_2 + L\frac{di_1(t)}{dt} = 0$$

$$L\frac{di_1(t)}{dt} + (R_1+R_2)\, i_1(t) = 0$$

$$i_1(t) = i_{1p}(t) + i_{1c}(t)$$

$i_{1p}(t)$ – the particular solution

$$i_{1p}(t) = 0$$

$i_{1c}(t)$ – the complementary solution

$$L\frac{di_{1c}(t)}{dt} + (R_1+R_2)\, i_{1c}(t) = 0 \qquad Ls_1 + R_1 + R_2 = 0 \qquad s_1 = -\frac{R_1+R_2}{L}$$

$$i_{1c} = A_1 e^{s_1 t}$$

at $t=0$: $\quad i_1(t) = i_{1p}(t) + i_{1c}(t)$

$$4 = 0 + A_1 e^0 \Rightarrow A_1 = 4$$

$$i_1(t) = 4e^{-1000t}\,A$$

$$i_2(t) = -i_1(t) = -4e^{-1000t}\,A$$