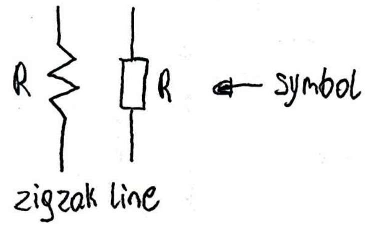
ELECTRICAL CIRCUITS 1 - CLASS NO. 01 (07.10.2024)

RESISTOR



R - resistance

* the letter symbol for resistance is R
* the unit of resistance is Ohm (Ω) - Greek letter Ω (omega)
* IV- equation:

$$v_R(t) = R \cdot i_R(t) = \frac{i_R(t)}{G} \quad \text{Ohm's Law}$$

* the invers of resistance is the conductance [S]

RESISTANCE

$$R = \rho \cdot \frac{L}{A}$$

L - the length of the conductor [m]
A - cross-sectional area of a conductor
ρ (rho) - the electrical resistivity of the material
[$\Omega \cdot m$] → [ohm-meters]

* the resistance of a given object depends on two factors:
  - what material it is made of
  - its shape

PROBLEM #1

Calculate the length of the copper wire with cross-sectional area: a) 1.5 mm^2 b) 2.5 mm^2 when its resistance is 1Ω.

$$R = \rho \cdot \frac{L}{A}$$

R - resistance [Ω]
ρ - electrical resistivity
L - the length
A - cross-sectional area

$\rho_{Cu} = 1.7 \cdot 10^{-8}\ \Omega \cdot m$

$$L = \frac{R \cdot A}{\rho}$$

a) $R = 1\Omega$, $A = 1.5 \cdot 10^{-6} m^2$, $\rho = 1.7 \cdot 10^{-8}\ \Omega \cdot m$

$$L = \frac{R \cdot A}{\rho} = \frac{1 \cdot 1.5 \cdot 10^{-6}}{1.7 \cdot 10^{-8}} = 88,24\ m$$

b) $R = 1\Omega$, $A = 2.5 \cdot 10^{-6} m^2$, $\rho = 1.7 \cdot 10^{-8}\ \Omega m$

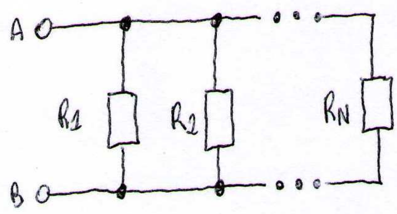
$$L = \frac{R \cdot A}{\rho} = \frac{1 \cdot 2.5 \cdot 10^{-6}}{1.7 \cdot 10^{-8}} = 147.06\ m$$

RESISTORS IN SERIES

$$R_{EQ} = R_1 + R_2 + \ldots + R_N$$

* the resistors are conected in series (betueen a pair of terminals A and B)
* the total (equivalent) resistance of resistors connected in series is equal to the sum of their individual resistances

RESISTORS IN PARALLEL

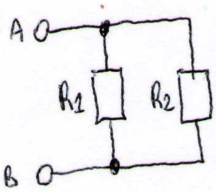


$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

$$G = \frac{1}{R} \qquad G_{EQ} = G_1 + G_2 + \ldots + G_N$$

* repricolar of the equivalent resistance of resistors connected in parallel is equal to the sum of the repricolars of the resistances

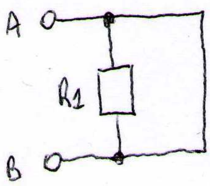
* if $R_1 = R_2 = \ldots = R_N = R \Rightarrow \frac{1}{R_{EQ}} = \underbrace{\frac{1}{R} + \frac{1}{R} + \ldots + \frac{1}{R}}_{N} = \frac{N}{R} \longrightarrow \boxed{R_{EQ} = \frac{R}{N}}$



$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} = \underbrace{\frac{R_2}{R_1 \cdot R_2} + \frac{R_1}{R_1 \cdot R_2} = \frac{R_1 + R_2}{R_1 \cdot R_2}}_{\text{we use common denominator}} \longrightarrow \boxed{R_{EQ} = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

* if $R_1 = R_2 = R \Rightarrow R_{EQ} = \frac{R \cdot R}{R + R} = \frac{R^2}{2R} = \frac{R}{2}$

* if $R_1 = R,\ R_2 = 0$

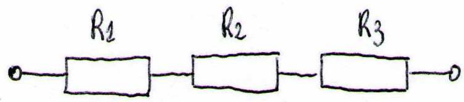


$$R_{EQ} = \frac{R_1 \cdot 0}{R_1 + 0} = \frac{0}{R_1} = 0$$

PROBLEM #2

Calculate the equivalent resistance of circuits shown below.

a)

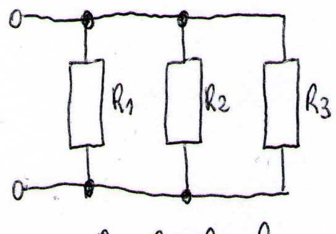


$R_1 = R_2 = R_3 = R$

$R_{EQ} = R_1 + R_2 + R_3 =$

$R_{EQ} = R_1 + R_2 + R_3 = R + R + R = \boxed{3R}$

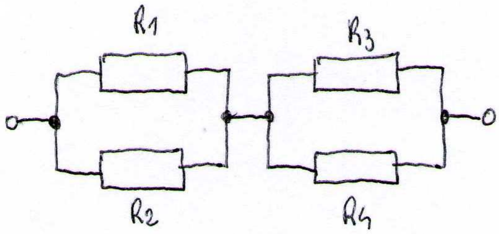
b)



$R_1 = R_2 = R_3 = R$

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R} \rightarrow \boxed{R_{EQ} = \frac{R}{3}}$$

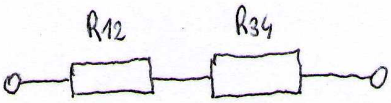
c)



$R_1 = R_2 = R_3 = R_4 = R$

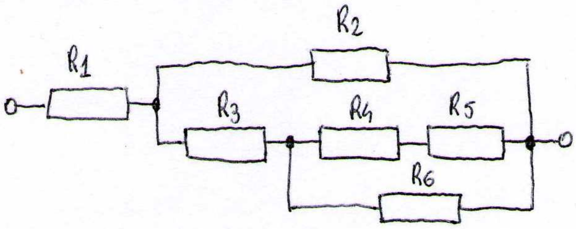
$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{R \cdot R}{R + R} = \frac{R}{2}$$

$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4} = \frac{R \cdot R}{R + R} = \frac{R}{2}$$



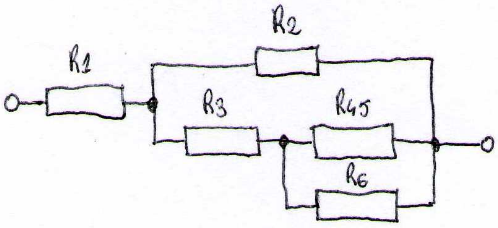
$$R_{EQ} = R_{12} + R_{34} = \frac{R}{2} + \frac{R}{2} = \boxed{R}$$

d)

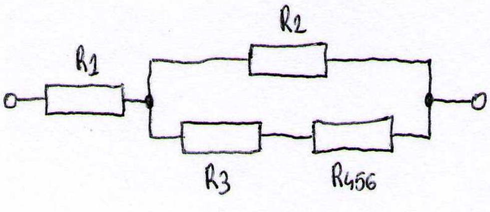


$$R_{45} = R_4 + R_5 = R + R = 2R$$

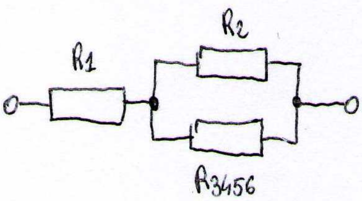
$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R$



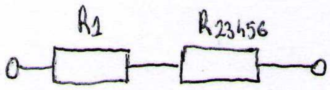
$$R_{456} = \frac{R_{45} \cdot R_6}{R_{45} + R_6} = \frac{2R \cdot R}{2R + R} = \frac{2R^{\not{2}}}{3\not{R}} = \frac{2}{3}R$$



$$R_{3456} = R_3 + R_{456} = R + \frac{2}{3}R = \frac{5}{3}R$$

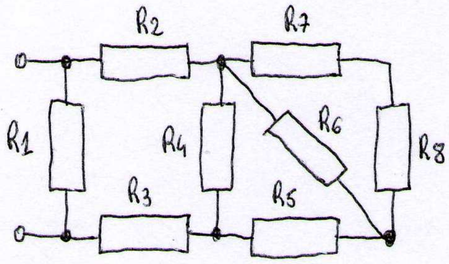


$$R_{23456} = \frac{R_2 \cdot R_{3456}}{R_2 + R_{3456}} = \frac{R \cdot \frac{5}{3}R}{R + \frac{5}{3}R} = \frac{\frac{5}{\not{3}}R^{\not{2}}}{\frac{8}{\not{3}}\not{R}} = \frac{5}{8}R$$



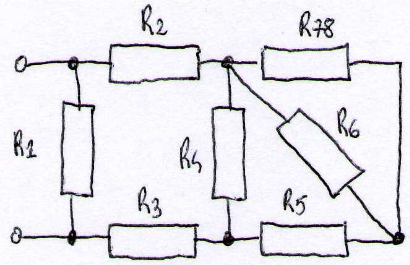
$$R_{EQ} = R_1 + R_{23456} = R + \frac{5}{8}R = \boxed{\frac{13}{8}R}$$

e)

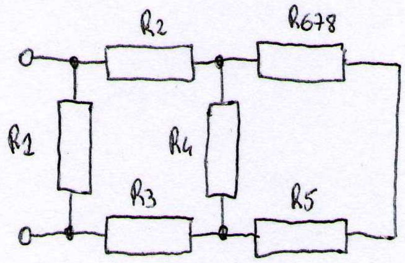


$R_1 = R_2 = \ldots = R_8 = R$

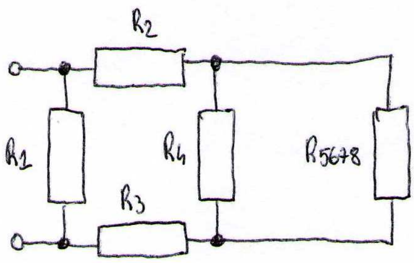
$$R_{78} = R_7 + R_8 = R + R = 2R$$



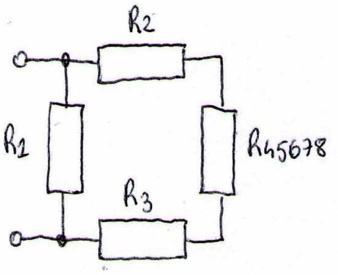
$$R_{678} = \frac{R_6 \cdot R_{78}}{R_6 + R_{78}} = \frac{R \cdot 2R}{R + 2R} = \frac{2R^2}{3R} = \frac{2}{3}R$$



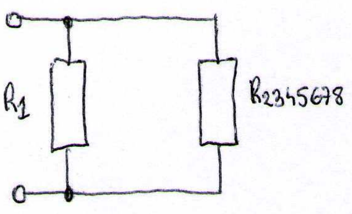
$$R_{5678} = R_5 + R_{678} = R + \frac{2}{3}R = \frac{5}{3}R$$



$$R_{45678} = \frac{R_4 \cdot R_{5678}}{R_4 + R_{5678}} = \frac{R \cdot \frac{5}{3}R}{R + \frac{5}{3}R} = \frac{\frac{5}{3}R^2}{\frac{8}{3}R} = \frac{5}{8}R$$

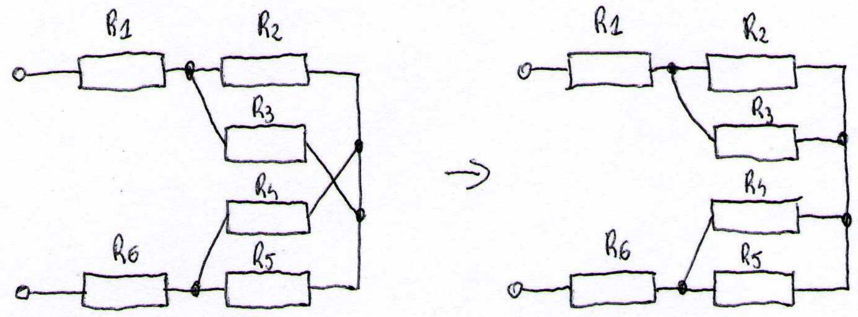


$$R_{2345678} = R_2 + R_3 + R_{45678} = R + R + \frac{5}{8}R = \frac{21}{8}R$$



$$R_{EQ} = \frac{R_1 \cdot R_{2345678}}{R_1 + R_{2345678}} = \frac{R \cdot \frac{21}{8}R}{R + \frac{21}{8}R} = \frac{\frac{21}{8}R^2}{\frac{29}{8}R} = \boxed{\frac{21}{29}R}$$

f)

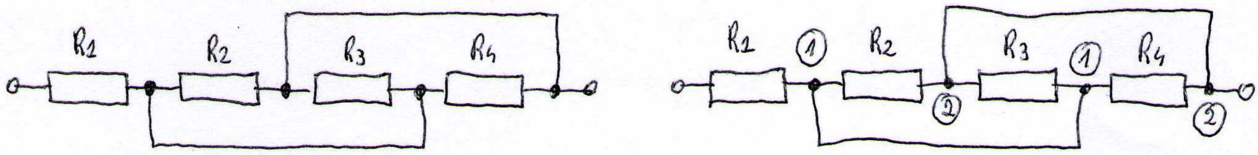


$R_1 = R_2 = \ldots = R_6 = R$

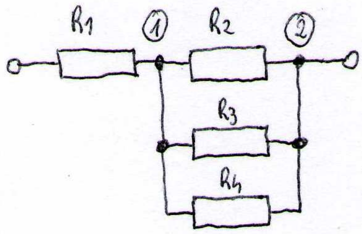
$$R_{EQ} = R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} + \frac{R_4 \cdot R_5}{R_4 + R_5} + R_6$$

$$= R + \frac{R \cdot R}{2R} + \frac{R \cdot R}{2R} + R = R + \frac{R}{2} + \frac{R}{2} + R = \boxed{3R}$$

g)

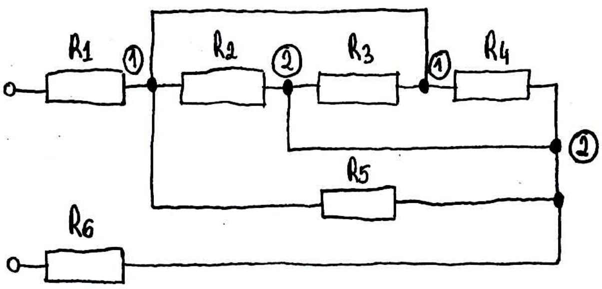


$R_1 = R_2 = R_3 = R_4 = R$

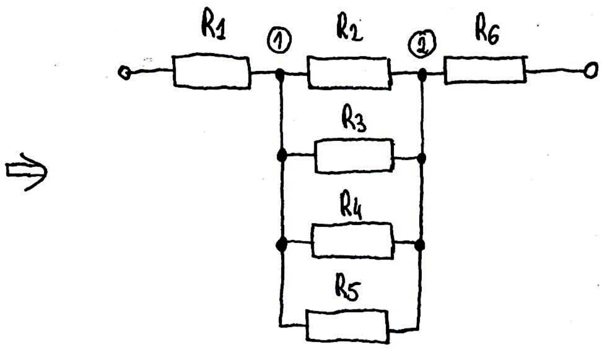


$$R_{EQ} = R + \frac{R}{3} = \boxed{\frac{4}{3}R}$$

h)



$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R$



$$R_{EQ} = R + \frac{R}{4} + R = \boxed{\frac{9}{4}R}$$