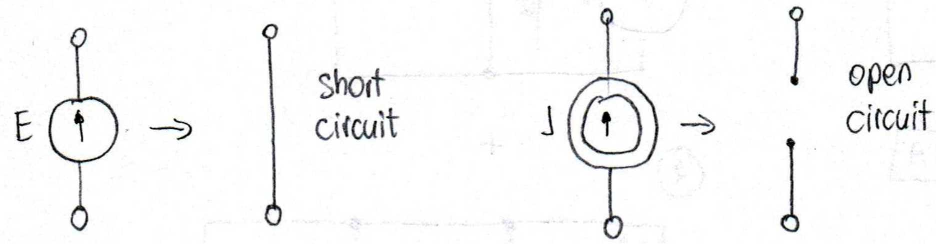
ELECTRICAL CIRCUITS 1 - CLASS NO. 04 (28.10.2024)

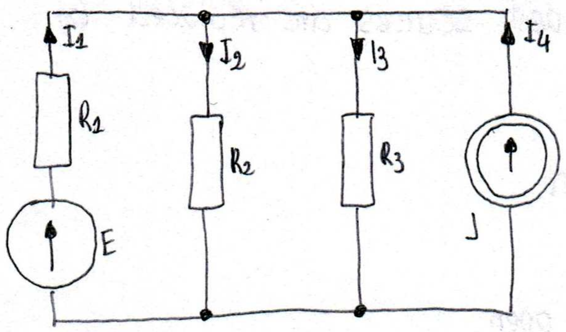
THE SUPERPOSITION THEOREM

* the current in any branch of linear circuit having more than one independent source equals the algebraic sum of the currents caused by each independent source acting alone where the all other independent sources are replaced by their internal resistances
* the voltage source is replaced with a short circuit
* the current source is replaced with an open circuit

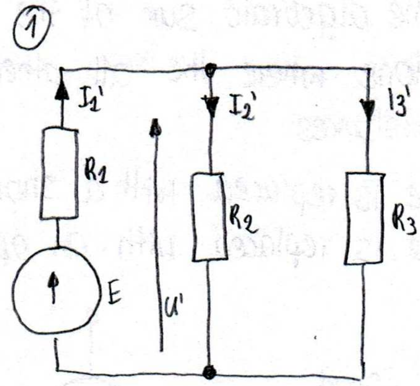


PROBLEM #1

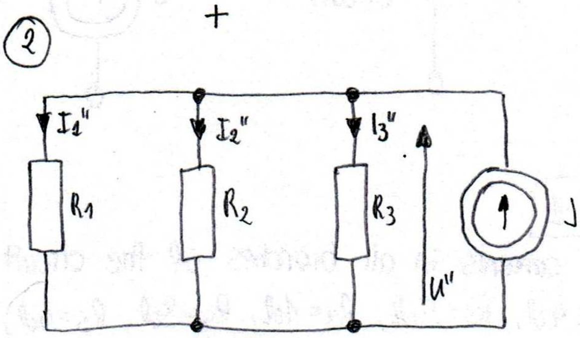
Calculate the currents in all branches of the circuit presented in the figure. Use the superposition theorem. $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 2\Omega$, $J = 1A$, $E = 4V$.



=



+



①
$$I_1' = \frac{E}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}} = \frac{4}{1 + \frac{2 \cdot 2}{2+2}} = \frac{4}{2} = \boxed{2\,A}$$

$$U' = I_1' \cdot \frac{R_2 \cdot R_3}{R_2 + R_3} = 2 \cdot \frac{2+2}{2+2} = 2\ V$$

$$I_2' = \frac{U}{R_2} = \frac{2}{2} = \boxed{1\ A} \qquad I_3' = \frac{U}{R_3} = \frac{2}{2} = \boxed{1\ A}$$

②
$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{2} = \frac{2}{1} \qquad R_{EQ} = 0.5\,\Omega \qquad U'' = J \cdot R_{EQ} = 1 \cdot 0.5 = 0.5\ V$$

$$I_1'' = \frac{U}{R_1} = \frac{0.5}{1} = \boxed{0.5\ A} \qquad I_2'' = \frac{U}{R_2} = \frac{0.5}{2} = \boxed{0.25A} \qquad I_3'' = \frac{U}{R_3} = \frac{0.5}{2} = \boxed{0.25\ A} \qquad I_4 = J = \boxed{1\ A}$$

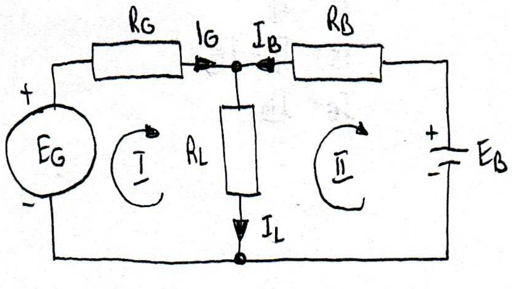
$$I_1 = I_1' - I_1'' = 2 - 0.5 = \boxed{1.5\ A} \qquad I_2 = I_2' + I_2'' = 1 + 0.25 = \boxed{1.25\ A} \qquad I_3 = I_3' + I_3'' = 1 + 0.25 = \boxed{1.25\ A}$$

THE LOOP-CURRENT METHOD

* the analysis is performed with the following sequence of steps:
  1. identify the loops (a loop - any closed path around a circuit)
     - the loops should be independent - at least one element in each loop should be not part of any other loop
  2. assign a loop current to each loop
     - a loop current is an imagined current flowing around a loop
     - the directions of the loop currents do not matter, but usually a consistent direction (clockwise or counterclockwise) is used
     - use Roman numerals for the loop currents numbering
     - use Arabic numerals for the branch currents numbering
  3. write KVL equations around each loop
  4. solve the resulting system of equations for all loop currents
  5. the branch current depends on the loop currents flowing in the branch

PROBLEM #2

Calculate the currents in all branches of the circuit presented in the figure. Use the Loop-Current method. $E_G = 16V$, $R_G = 0.2\Omega$, $E_B = 12.8V$, $R_B = 0.1\Omega$, $R_L = 1\Omega$.



$$\begin{cases} I_I(R_G + R_L) - I_{II}R_L = E_G \\ I_{II}(R_L + R_B) - I_I R_L = -E_B \end{cases}$$

$$\begin{cases} I_I(0.2+1) - I_{II}\cdot 1 = 16 \\ I_{II}(1+0.1) - I_I \cdot 1 = -12.8 \end{cases}$$

$$\begin{cases} 1.2 I_I - I_{II} = 16 \\ -I_I + 1.1 I_{II} = -12.8 \end{cases}$$

$$-I_{II} = 16 - 1.2 I_I \quad |\cdot(-1)$$

$$I_{II} = 1.2 I_I - 16$$

$$-I_I + 1.1(1.2 I_I - 16) = -12.8$$

$$-I_I + 1.32 I_I - 17.6 = -12.8$$

$$0.32 I_I = 4.8 \quad |:(0.32)$$

$$\boxed{I_I = 15A}$$

$$I_{II} = 1.2 I_I - 16$$

$$I_{II} = 1.2 \cdot 15 - 16$$

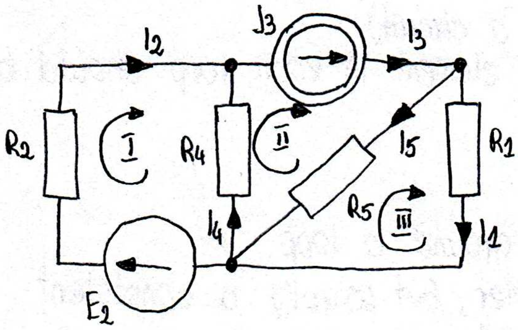
$$I_{II} = 18 - 16$$

$$\boxed{I_{II} = 2A}$$

$I_G = I_I = \boxed{15A}$ $\quad I_L = I_I - I_{II} = 15 - 2 = \boxed{13A}$ $\quad I_B = -I_{II} = \boxed{-2A}$

PROBLEM #3

Calculate the currents in all branches of the circuit presented in the figure. Use the Loop-Current Method. $R_1=4\Omega$, $R_2=3\Omega$, $R_4=2\Omega$, $R_5=6\Omega$, $J_3=5A$, $E_2=10V$.



$$\begin{cases}(R_2+R_4)\,I_I-R_4\,I_{II}=E_2\\ I_{II}=J_3\\ (R_1+R_5)\,I_{III}-R_5\,I_{II}=0\end{cases}$$

$$\boxed{I_{II}=5A}$$

$(3+2)\,I_I-2\cdot5=10$

$5I_I=10+10$

$$\boxed{I_I=\frac{20}{5}=4A}$$

$(4+6)\,I_{III}-6\cdot5=0$

$10\,I_{III}=30$

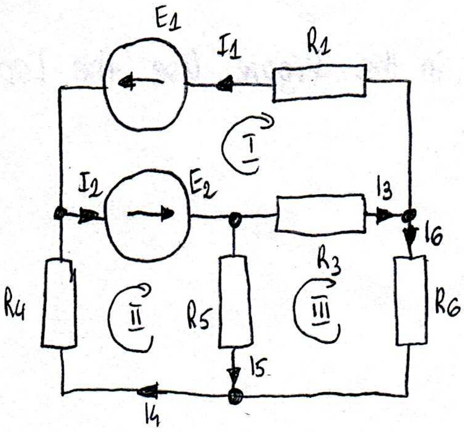
$$\boxed{I_{III}=\frac{30}{10}=3A}$$

$\boxed{I_1=I_{III}=3A}$ $\boxed{I_2=I_I=4A}$ $\boxed{I_3=J_3=5A}$

$\boxed{I_4=I_{II}-I_I=5-4=1A}$ $\boxed{I_5=I_{II}-I_{III}=5-3=2A}$

PROBLEM #4

Write the equations according to the Loop-Current Method for the circuit shown in the figure.



$$\begin{cases}I_I\,(R_1+R_3)-I_{III}\,R_3=-E_1-E_2\\ I_{II}\,(R_4+R_5)-I_{III}\,R_5=E_2\\ I_{III}\,(R_3+R_5+R_6)-I_I\,R_3-I_{II}\,R_5=0\end{cases}$$

$I_1=-I_I$

$I_2=I_{II}-I_I$

$I_3=I_{III}-I_I$

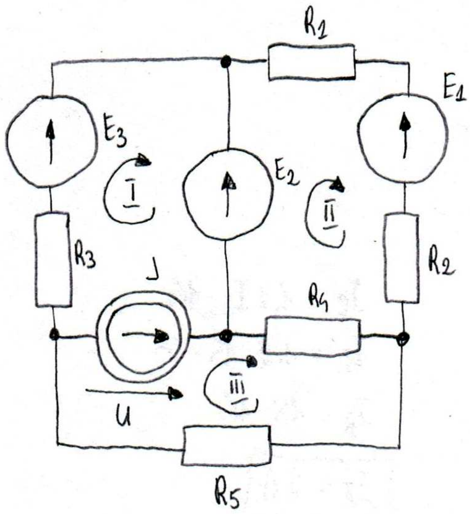
$I_4=I_{II}$

$I_5=I_{II}-I_{III}$

$I_6=I_{III}$

PROBLEM #5

Write the equations according to the Loop-Current Method for the circuit shown in the figure.



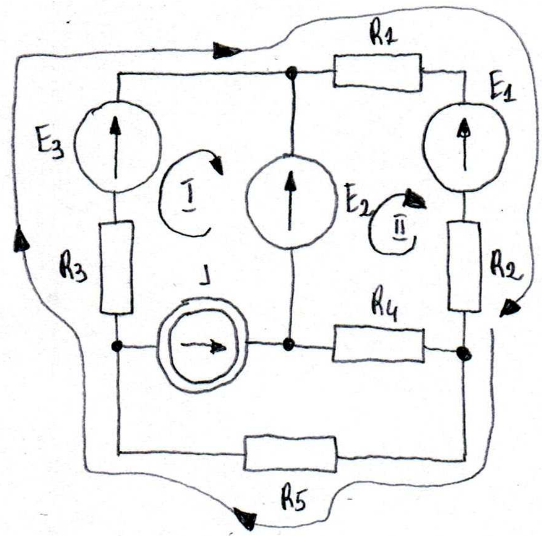
Method 1

* we assume that there is a voltage drop U on the current source

$$\begin{cases}I_I\cdot R_3=E_3-E_2-U\\ I_{II}\,(R_1+R_2+R_4)-I_{III}\,R_4=E_2-E_1\\ I_{III}\,(R_4+R_5)-I_{II}\,R_4=U\\ J=I_{III}-I_I\end{cases}$$

Method 2

* we set the loop current circuit in such a way that only one loop current flows through the current source



$$
\begin{cases}
I_{I} = -J \\
I_{II}(R_1+R_2+R_4) + I_{III}(R_1+R_2) = E_2 - E_1 ; \\
I_{III}(R_1+R_2+R_3+R_5) + I_{I} R_3 + I_{II}(R_1+R_2) = -E_1 + E_3
\end{cases}
$$