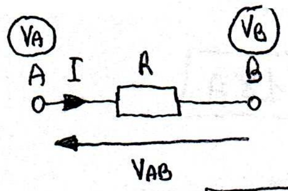
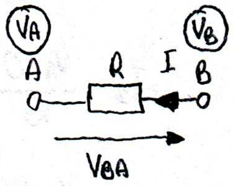
ELECTRICAL CIRCUITS 1 - CLASS NO. 05 (04.11.2024)

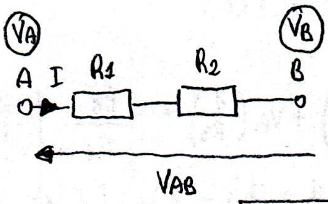
REMARKS: voltage - electric potential difference



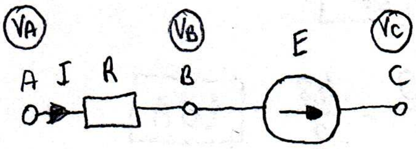
$$I = \frac{V_{AB}}{R} = \boxed{\frac{V_A - V_B}{R}}$$



$$I = \frac{V_{BA}}{R} = \boxed{\frac{V_B - V_A}{R}}$$

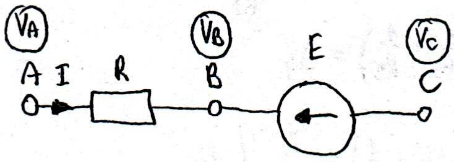


$$I = \frac{V_{AB}}{R_1 + R_2} = \boxed{\frac{V_A - V_B}{R_1 + R_2}}$$



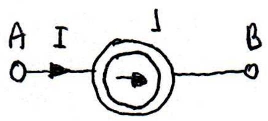
$$I = \frac{V_A - V_B}{R} \qquad E = V_C - V_B \Rightarrow V_B = V_C - E$$

$$I = \boxed{\frac{V_A - V_C + E}{R}}$$

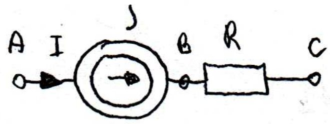


$$I = \frac{V_A - V_B}{R} \qquad E = V_B - V_C \Rightarrow V_B = E + V_C$$

$$I = \boxed{\frac{V_A - V_C - E}{R}}$$



$$\boxed{I = J}$$



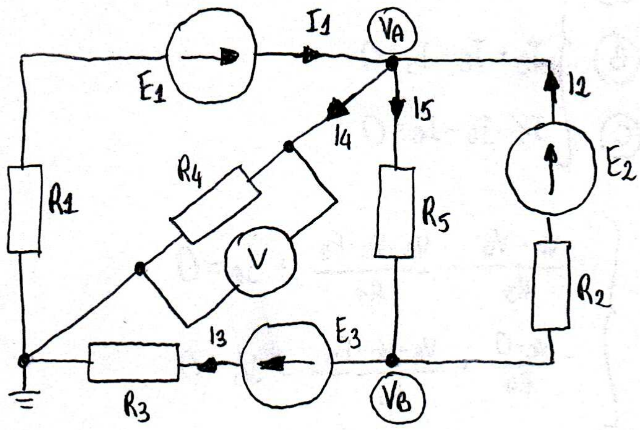
$$\boxed{I = J}$$

THE NODE-VOLTAGE METHOD

* the method is based on Kirchhoff's Current Law
* in this method we select one of the nodes in the circuit to be the reference node (the ground node)
* the potential of the ground node is defined to be 0V
* all the other node voltages are measured with respect to this one reference node
* the sequence of the steps:
  - identify all nodes
  - choose a reference node
  - assign voltage variables to the others nodes (node voltages)
  - write a KCL equation for each node (use Ohm's Law)
  - solve the resulting system of equations for all node voltages
  - calculate currents using Ohm's Law

PROBLEM #1

Calculate the voltmeter reading in the circuit shown in the figure. Use the Node-Voltage Method. $E_1 = 12V$, $E_2 = E_3 = 6V$, $R_1 = 100\Omega$, $R_2 = 380\Omega$, $R_3 = 330\Omega$, $R_4 = 150\Omega$, $R_5 = 120\Omega$, $R_V = \infty$.



$$\begin{cases} (A) & I_1 + I_2 - I_4 - I_5 = 0 \\ (B) & I_5 - I_2 - I_3 = 0 \end{cases}$$

$$I_1 = \frac{0 - V_A + E_1}{R_1} \qquad I_2 = \frac{V_B - V_A + E_2}{R_2}$$

$$I_3 = \frac{V_B - 0 + E_3}{R_3} \qquad I_4 = \frac{V_A - 0}{R_4}$$

$$I_5 = \frac{V_A - V_B}{R_5}$$

$$\begin{cases} (A) & \frac{0 - V_A + E_1}{R_1} + \frac{V_B - V_A + E_2}{R_2} - \frac{V_A - 0}{R_4} - \frac{V_A - V_B}{R_5} = 0 \\ (B) & \frac{V_A - V_B}{R_5} - \frac{V_B - V_A + E_2}{R_2} - \frac{V_B - 0 + E_3}{R_3} = 0 \end{cases}$$

$$\begin{cases} -V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) + V_B\left(\frac{1}{R_2} + \frac{1}{R_5}\right) + \frac{E_1}{R_1} + \frac{E_2}{R_2} = 0 \quad | \cdot (-1) \\ -V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) + V_A\left(\frac{1}{R_2} + \frac{1}{R_5}\right) - \frac{E_2}{R_2} - \frac{E_3}{R_3} = 0 \quad | \cdot (-1) \end{cases}$$

$$\begin{cases} V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) - V_B\left(\frac{1}{R_2} + \frac{1}{R_5}\right) = \frac{E_1}{R_1} + \frac{E_2}{R_2} \\ -V_A\left(\frac{1}{R_2} + \frac{1}{R_5}\right) + V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) = -\frac{E_2}{R_2} - \frac{E_3}{R_3} \end{cases}$$

$$\begin{cases} V_A\left(\frac{1}{100} + \frac{1}{380} + \frac{1}{150} + \frac{1}{120}\right) - V_B\left(\frac{1}{380} + \frac{1}{120}\right) = \frac{12}{100} + \frac{6}{380} \\ -V_A\left(\frac{1}{380} + \frac{1}{120}\right) + V_B\left(\frac{1}{380} + \frac{1}{330} + \frac{1}{120}\right) = -\frac{6}{380} - \frac{6}{330} \end{cases}$$

$$\begin{cases} 0.0276\, V_A - 0.0109\, V_B = 0.2739 \\ -0.0109\, V_A + 0.0139\, V_B = -0.0336 \end{cases}$$

$$W = \begin{vmatrix} 0.0276 & -0.0109 \\ -0.0109 & 0.0139 \end{vmatrix} = 2.6483 \cdot 10^{-4}$$

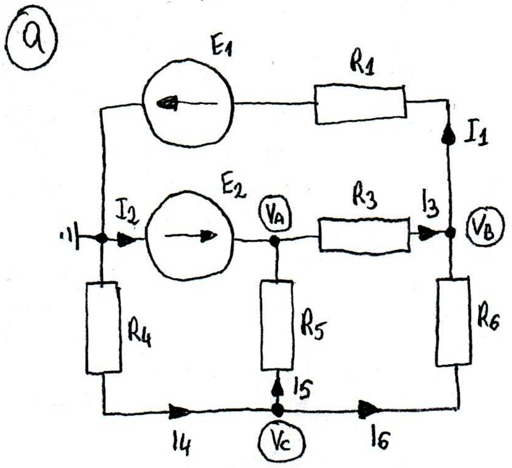
$$V_A = \frac{W_A}{W} = \frac{3.44097 \cdot 10^{-3}}{2.6483 \cdot 10^{-4}} = \boxed{13\ V}$$

$$W_A = \begin{vmatrix} 0.2739 & -0.0109 \\ -0.0336 & 0.0139 \end{vmatrix} = 3.44097 \cdot 10^{-3}$$

PROBLEM #2

Write the equations according to the Node-Voltage Method for the circuits shown in the figures

(a)



$$V_A \left\{ \; V_A = E_2 \right.$$

$$V_B \left\{ \; I_3 + I_6 - I_1 = 0 \right.$$

$$V_C \left\{ \; I_4 - I_5 - I_6 = 0 \right.$$

$$I_1 = \frac{V_B - 0 + E_1}{R_1} \qquad I_5 = \frac{V_C - V_A}{R_5}$$

$$I_3 = \frac{V_A - V_B}{R_3} \qquad I_6 = \frac{V_C - V_B}{R_6}$$

$$I_4 = \frac{0 - V_C}{R_4}$$

$$\frac{\overset{E_2}{V_A} - V_B}{R_3} + \frac{V_C - V_B}{R_6} - \frac{V_B + E_1}{R_1} = 0$$

$$\frac{E_2}{R_3} - \frac{V_B}{R_3} + \frac{V_C}{R_6} - \frac{V_B}{R_6} - \frac{V_B}{R_1} - \frac{E_1}{R_1} = 0$$

$$-V_B\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_6}\right) + V_C\left(\frac{1}{R_6}\right) = \frac{E_1}{R_1} - \frac{E_2}{R_3} \quad \Big| \cdot (-1)$$

$$\boxed{V_B\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_6}\right) - V_C\left(\frac{1}{R_6}\right) = -\frac{E_1}{R_1} + \frac{E_2}{R_3}}$$

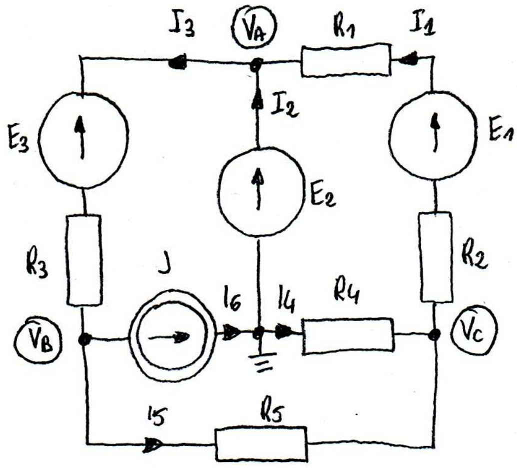
$$\frac{0 - V_C}{R_4} - \frac{V_C - \overset{E_2}{V_A}}{R_5} - \frac{V_C - V_B}{R_6} = 0$$

$$-\frac{V_C}{R_4} - \frac{V_C}{R_5} + \frac{E_2}{R_5} - \frac{V_C}{R_6} + \frac{V_B}{R_6} = 0$$

$$V_B\left(\frac{1}{R_6}\right) - V_C\left(\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}\right) = -\frac{E_2}{R_5} \quad \Big| \cdot (-1)$$

$$\boxed{-V_B\left(\frac{1}{R_6}\right) + V_C\left(\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}\right) = \frac{E_2}{R_5}}$$

(b)



$$V_A \left\{ \; V_A = E_2 \right.$$

$$V_B \left\{ \; I_3 - I_5 - I_6 = 0 \right.$$

$$V_C \left\{ \; I_4 + I_5 - I_1 = 0 \right.$$

$$I_1 = \frac{V_C - V_A + E_1}{R_1 + R_2} \qquad I_5 = \frac{V_B - V_C}{R_5}$$

$$I_3 = \frac{V_A - V_B - E_3}{R_3} \qquad I_6 = J$$

$$I_4 = \frac{0 - V_C}{R_4}$$

$$\frac{V_A - V_B - E_3}{R_3} - \frac{V_B - V_C}{R_5} - J = 0$$

$$\frac{E_2}{R_3} - \frac{V_B}{R_3} - \frac{E_3}{R_3} - \frac{V_B}{R_5} + \frac{V_C}{R_5} - J = 0$$

$$\boxed{V_B\left(\frac{1}{R_3} + \frac{1}{R_5}\right) - V_C\left(\frac{1}{R_5}\right) = \frac{E_2}{R_3} - \frac{E_3}{R_3} - J}$$

$$\frac{0 - V_C}{R_4} + \frac{V_B - V_C}{R_5} - \frac{V_C - V_A + E_1}{R_1 + R_2} = 0$$

$$-\frac{V_C}{R_4} + \frac{V_B}{R_5} - \frac{V_C}{R_5} - \frac{V_C}{R_1 + R_2} + \frac{E_2}{R_1 + R_2} - \frac{E_1}{R_1 + R_2} = 0$$

$$\boxed{-V_B\left(\frac{1}{R_5}\right) + V_C\left(\frac{1}{R_1 + R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) = -\frac{E_1}{R_1 + R_2} + \frac{E_2}{R_1 + R_2}}$$