ELECTRICAL CIRCUITS 1 - CLASS NO. 11 (13.01.2025)

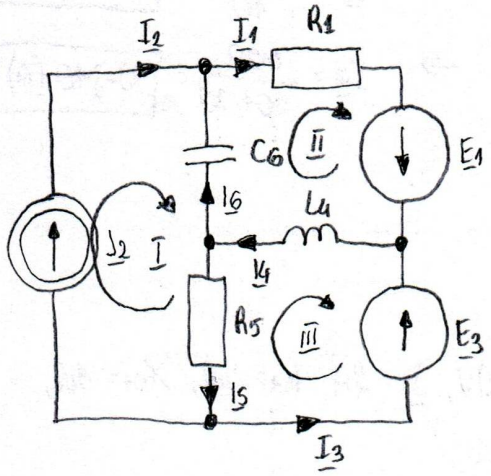
LOOP-CURRENT METHOD IN AC CIRCUITS

* the analysis is perfomed with the following sequence of steps
  1. identify the loops
  2. assign a loop current to each loop
  3. write KVL equation around each loop
  4. solve the resulting system of equations for all loop currents
  5. the branch current depends on the loop currents flowing in the branch

PROBLEM #1

Calculate the currents in all branches of the circuit shown in the figure using the Loop-Current Method.

$R_1 = 20\Omega$, $X_{L4} = 20\Omega$, $R_5 = 10\Omega$, $X_{C6} = 20\Omega$, $\underline{E}_1 = j20\,V$, $E_3 = 20\,V$, $\underline{J}_2 = 10A$.



$$\begin{cases} \underline{I}_I = \underline{J}_2 = 10A \\ \underline{I}_{II}(R_1 + jX_{L4} - jX_{C6}) - (-jX_{C6})\cdot\underline{I}_I - (jX_{L4})\cdot\underline{I}_{III} = \underline{E}_1 \\ \underline{I}_{III}(R_5 + jX_{L4}) - \underline{I}_I(R_5) - \underline{I}_{II}(jX_{L4}) = -\underline{E}_3 \end{cases}$$

$$\begin{cases} 10\cdot j20 + \underline{I}_{II}(20 + j20 - j20) - \underline{I}_{III}(j20) = j20 \\ -10\cdot 10 - \underline{I}_{II}(j20) + \underline{I}_{III}(10 + j20) = -20 \end{cases}$$

$$\begin{cases} \underline{I}_{II}\cdot 20 - \underline{I}_{III}\cdot j20 = -j180 \\ -\underline{I}_{II}\cdot j20 + \underline{I}_{III}(10 + j20) = 80 \end{cases}$$

Cramer's rule

$$W = \begin{vmatrix} 20 & -j20 \\ -j20 & 10+j20 \end{vmatrix} = 200 + j400 + 400 = 600 + j400$$

$$W_{II} = \begin{vmatrix} -j180 & -j20 \\ 80 & 10+j20 \end{vmatrix} = -j1800 + 3600 + j1600 = 3600 - j200$$

$$W_{III} = \begin{vmatrix} 20 & -j180 \\ -j20 & 80 \end{vmatrix} = 1600 + 3600 = 5200$$

$$\underline{I}_{II} = \frac{W_{II}}{W} = \frac{3600 - j200}{600 + j400} = 4 - j3\ (A)$$

$$\underline{I}_{III} = \frac{W_{III}}{W} = \frac{5200}{600 + j400} = 6 - j4\ (A)$$

$\underline{I}_1 = \underline{I}_{II} = \boxed{4 - j3\ (A)}$

$\underline{I}_2 = \underline{J}_2 = \boxed{10\ (A)}$

$\underline{I}_3 = -\underline{I}_{III} = \boxed{-6 + j4\ (A)}$

$\underline{I}_4 = \underline{I}_{II} - \underline{I}_{III} = 4 - j3 - 6 + j4 = \boxed{-2 + 1j\ (A)}$

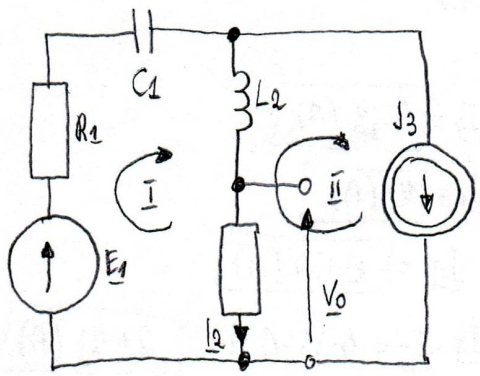
$\underline{I}_5 = \underline{I}_I - \underline{I}_{III} = 10 - 6 + j4 = \boxed{4 + j4\ (A)}$

$I_6 = \underline{I}_{II} - \underline{I}_I = 4 - j3 - 10 = \boxed{-6 - j3\ (A)}$

PROBLEM #2

Determine the voltage Vo using the Loop-Current Method.

$R_1 = 2\Omega$, $X_{C1} = 1\Omega$, $X_{L2} = 2\Omega$, $R_2 = 4\Omega$, $\underline{E}_1 = 12\,V$, $\underline{J}_3 = 2A$.



$\underline{V}_o = R_2 \cdot \underline{I}_2$

$\underline{I}_{II} = \underline{J}_3 = 2A$

$\underline{I}_I (R_1 - jX_{C1} + jX_{L2} + R_2) - \underline{I}_{II}(R_2 + jX_{L2}) = \underline{E}_1$

$\underline{I}_I (2 - j1 + j2 + 4) - 2(4 + j2) = 12$

$(6+j)\,\underline{I}_I = 12 + 2(4+j2)$

$(6+j)\,\underline{I}_I = 12 + 8 + j4$

$(6+j)\,\underline{I}_I = 20 + j4$

$\underline{I}_I = \dfrac{20+j4}{6+j} = (3.3514 + j\,0.1081)\,A$

$\underline{I}_2 = \underline{I}_I - \underline{I}_{II}$

$\underline{I}_2 = 3.3514 + j\,0.1081 - 2$

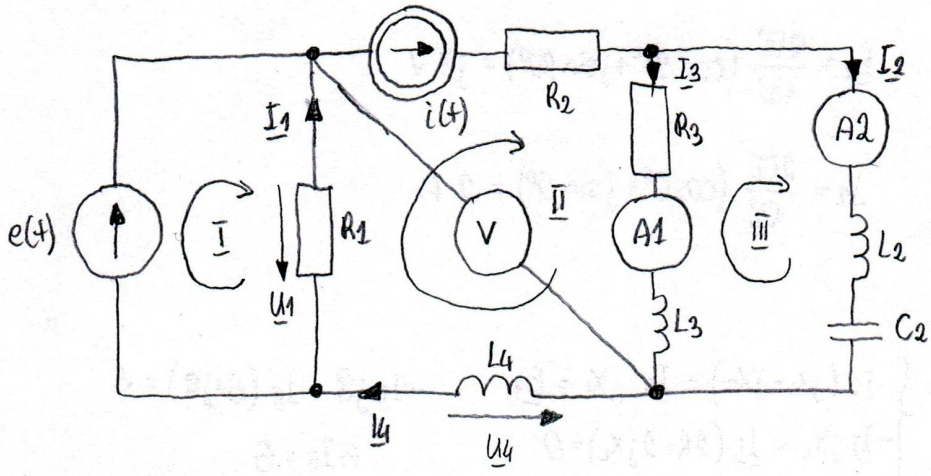
$\underline{I}_2 = (1.3514 + j\,0.1081)\,A$

$\underline{V}_o = R_2 \cdot \underline{I}_2 = 4 \cdot (1.3514 + j\,0.1081) = \boxed{(5.4054 + j\,0.4324)\,V}$

PROBLEM #3

Calculate the meter readings in the circuit shown below using the Loop-Current Method.

$e(t) = 100\sqrt{2}\sin(\omega t)\,V$, $i(t) = 5\sqrt{2}\cos(\omega t)\,A$, $R_1 = R_2 = R_3 = X_{L2} = X_{L3} = X_{L4} = X_{C2} = 10\Omega$.



$\underline{E} = \dfrac{100\sqrt{2}}{\sqrt{2}}(\cos 0° + j\sin 0°) = 100\,V$

$i(t) = 5\sqrt{2}\sin(\omega t + 90°)$

$\underline{I} = \dfrac{5\sqrt{2}}{\sqrt{2}}(\cos 90° + j\sin 90°) = j5\,A$

$$\begin{cases} \underline{I}_I R_1 - \underline{I}_{II} R_1 = \underline{E} \\ \underline{I}_{II} = \underline{J} = j5\,A \\ -\underline{I}_{II}(R_3 + jX_{L3}) + \underline{I}_{III}(R_3 + jX_{L3} + jX_{L2} - jX_{C2}) = 0 \end{cases}$$

$$\begin{cases} 10\,\underline{I}_I - 10\,\underline{I}_{II} = 100 \\ -\underline{I}_{II}(10 + j10) + \underline{I}_{III}(10 + j10 + j10 - j10) = 0 \end{cases}$$

$10\,\underline{I}_{I} - 10 \cdot j5 = 100$

$10\,\underline{I}_{I} = 100 + j50 \;/:10$

$\underline{I}_{I} = \boxed{(10+j5)\ A}$

$-j5\,(10+j10) + \underline{I}_{III}\,(10+j10) = 0$

$\underline{I}_{III}\,(10+j10) = -50 + j50$

$\underline{I}_{III} = \dfrac{-50+j50}{10+j10} = \boxed{(j5)\ A}$

$\underline{I}_{II} = \boxed{(j5)\ A}$

$\underline{I}_{A1} = \underline{I}_{II} - \underline{I}_{III} = j5 - j5 = 0\ A \rightarrow \boxed{I_{A1} = 0\,A}$

$\underline{I}_{A2} = \underline{I}_{III} = j5\,A = 5e^{j90°}\,A \rightarrow \boxed{I_{A2} = 5\,A}$

$\underline{U}_{V} = \underline{U}_{1} + \underline{U}_{4} = (\underline{I}_{II} - \underline{I}_{I}) \cdot R_1 + \underline{I}_{II} \cdot jX_{L4} = (j8 - 10 - j5) \cdot 10 + j5 \cdot j10 = -100 - 50 = -150\ V$

$\underline{U}_{V} = 150e^{j180°} \rightarrow \boxed{U_V = 150\,V}$