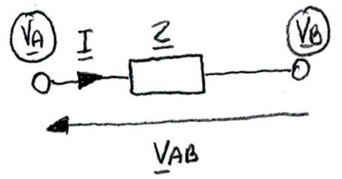
ELECTRICAL CIRCUITS 1 - CLASS NO. 12 (20.01.2025)

NODE-VOLTAGE METHOD IN AC CIRCUITS

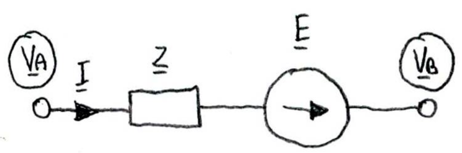
* the sequence of the steps:
  - identify all nodes
  - choose a reference node (the ground node) with potential equal to 0 V (which one? choose the node at the ideal voltage source or the one to which the most branches are connected)
  - assign voltage variables to the other nodes (node voltages)
  - write a KCL equation for each node
  - apply Ohm's law to any current
  - solve the resulting system of equations for all node voltages
  - calculate currents using Ohm's Law

* currents and Ohm's Law

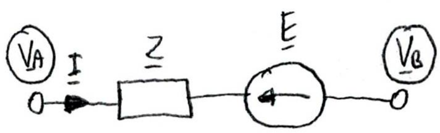


$$\underline{I} = \frac{\underline{V}_{AB}}{\underline{Z}} = \boxed{\frac{\underline{V}_A - \underline{V}_B}{\underline{Z}}}$$

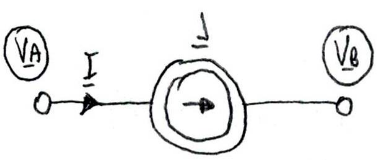
$$\underline{I} = \boxed{\frac{\underline{V}_A - \underline{V}_B}{R + jX_L - jX_C}}$$



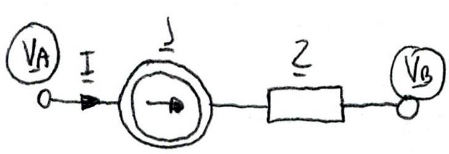
$$\underline{I} = \boxed{\frac{\underline{V}_A - \underline{V}_B + \underline{E}}{\underline{Z}}}$$



$$\underline{I} = \boxed{\frac{\underline{V}_A - \underline{V}_B - \underline{E}}{\underline{Z}}}$$



$$\boxed{\underline{I} = \underline{J}}$$



$$\boxed{\underline{I} = \underline{J}}$$

PROBLEM #1

Calculate the meter readings in the circuit shown in the figure using the Node-Voltage method. The parameters are as follows: $e(t) = 141.4 \sin(1000t)$ V, $R_1 = R_2 = R_3 = 50\Omega$, $L_1 = L_2 = 100$ mH, $C_1 = 20\,\mu F$, $C_3 = 10\,\mu F$.

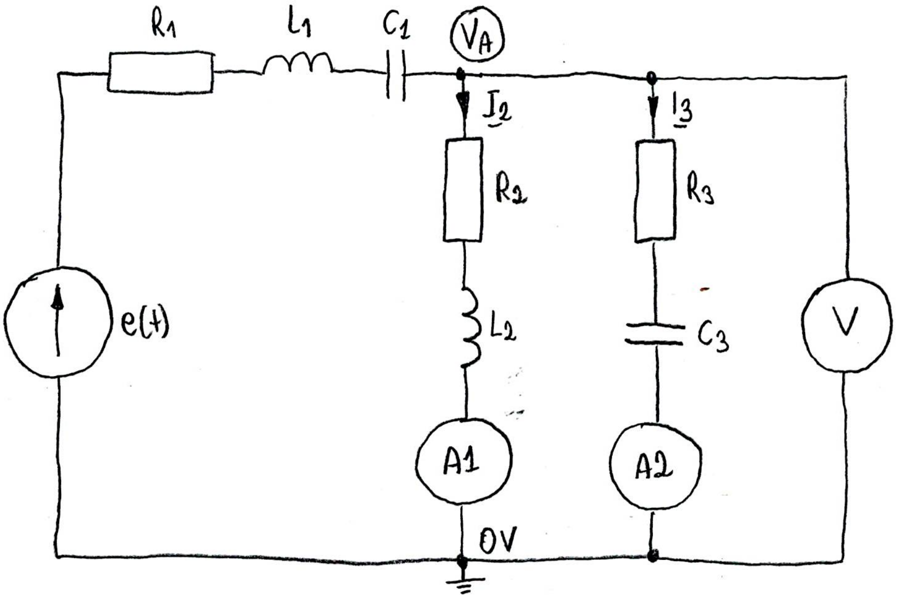
$$\underline{E} = \frac{141.4}{\sqrt{2}}(\cos 0^\circ + j \sin 0^\circ) = 100\,V$$

$$X_{L1} = \omega L_1 = 1000 \cdot 0.1 = 100\,\Omega$$

$$X_{L2} = \omega L_2 = 1000 \cdot 0.1 = 100\,\Omega$$

$$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{1000 \cdot 20 \cdot 10^{-6}} = 50\,\Omega$$

$$X_{C3} = \frac{1}{\omega C_3} = \frac{1}{1000 \cdot 10^{-6}} = 100\,\Omega$$



$$\underline{I}_1 - \underline{I}_2 - \underline{I}_3 = 0 \qquad \underline{I}_1 = \frac{0 - \underline{V}_A + \underline{E}}{R_1 + jX_{L1} - jX_{C1}} \qquad \underline{I}_2 = \frac{\underline{V}_A - 0}{R_2 + jX_{L2}} \qquad \underline{I}_3 = \frac{\underline{V}_A - 0}{R_3 - jX_{C3}}$$

$$\frac{\underline{E} - \underline{V}_A}{R_1 + jX_{L1} - jX_{C1}} - \frac{\underline{V}_A}{R_2 + jX_{L2}} - \frac{\underline{V}_A}{R_3 - jX_{C3}} = 0 \;\Big|\cdot(-1)$$

$$\frac{\underline{V}_A - \underline{E}}{R_1 + jX_{L1} - jX_{C1}} + \frac{\underline{V}_A}{R_2 + jX_{L2}} + \frac{\underline{V}_A}{R_3 - jX_{C3}} = 0$$

$$\underline{V}_A\left(\frac{1}{R_1 + jX_{L1} - jX_{C1}} + \frac{1}{R_2 + jX_{L2}} + \frac{1}{R_3 - jX_{C3}}\right) = \frac{\underline{E}}{R_1 + jX_{L1} - jX_{C1}}$$

$$\underline{V}_A\left(\frac{1}{50 + j100 - j50} + \frac{1}{50 + j100} + \frac{1}{50 - j100}\right) = \frac{100}{50 + j100 - j50}$$

$$\underline{V}_A\left(\frac{1}{50 + j50} + \frac{1}{50 + j100} + \frac{1}{50 - j100}\right) = \frac{100}{50 + j50}$$

$$\underline{V}_A(0.01 - j0.01 + 0.004 - \cancel{j0.008} + 0.004 + \cancel{j0.008}) = 1 - j1$$

$$\underline{V}_A(0.018 - j0.01) = 1 - j1$$

$$\underline{V}_A = \frac{1 - j1}{0.018 - j0.01} = (66.04 - j18.87)\,V$$

$$\underline{U}_V = \underline{V}_A - 0 = (66.04 - j18.87)\,V$$

$$|\underline{U}_V| = \boxed{68.68\,V}$$

$$\underline{I}_2 = \frac{\underline{V}_A}{R_2 + jX_{L2}} = \frac{66.04 - j18.87}{50 + j100} =$$

$$= (0.11 - j0.6)\,A$$

$$|\underline{I}_2| = I_{A1} = \boxed{0.61\,A}$$

$$\underline{I}_3 = \frac{\underline{V}_A}{R_3 - jX_{C3}} = \frac{66.04 - j18.87}{50 - j100} =$$

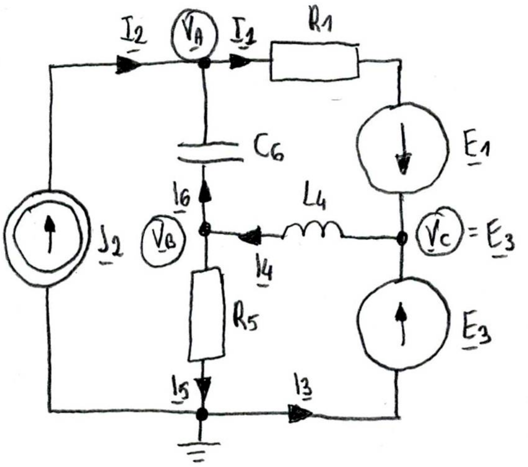
$$= (0.42 + j0.45)\,A$$

$$|\underline{I}_3| = I_{A2} = \boxed{0.61\,A}$$

PROBLEM #2

Calculate the currents in all branches of the circuit shown in the figure using the Node-Voltage Method. The parameters are as follows:

$R_1 = 20\,\Omega$, $X_{L4} = 20\,\Omega$, $R_5 = 10\,\Omega$, $X_{C6} = 20\,\Omega$, $\underline{E}_1 = j20\,V$, $\underline{E}_3 = 20\,V$, $\underline{J}_2 = 10\,A$



$$V_A:\ \underline{I}_2 + \underline{I}_6 - \underline{I}_1 = 0$$
$$V_B:\ \underline{I}_4 - \underline{I}_5 - \underline{I}_6 = 0$$
$$V_C:\ V_C = \underline{E}_3$$

$$\underline{I}_1 = \frac{V_A - V_C + E_1}{R_1} \qquad \underline{I}_2 = \underline{J}_2 \qquad \underline{I}_4 = \frac{V_C - V_B}{jX_{L4}}$$

$$\underline{I}_5 = \frac{V_B - 0}{R_5} \qquad \underline{I}_6 = \frac{V_B - V_A}{-jX_{C6}}$$

$$\begin{cases} \underline{J}_2 + \dfrac{V_B - V_A}{-jX_{C6}} - \dfrac{V_A - V_C + E_1}{R_1} = 0 \\ \dfrac{V_C - V_B}{jX_{L4}} - \dfrac{V_B}{R_5} - \dfrac{V_B - V_A}{-jX_{C6}} = 0 \end{cases}$$

$$\begin{cases} V_A\left(\dfrac{1}{-j20} + \dfrac{1}{20}\right) - V_B\left(\dfrac{1}{-j20}\right) = \dfrac{20}{20} - \dfrac{j20}{20} + 10 \\ -V_A\left(\dfrac{1}{-j20}\right) + V_B\left(\dfrac{1}{j20} + \dfrac{1}{10} + \dfrac{1}{-j20}\right) = \dfrac{20}{j20} \end{cases}$$

$$\begin{cases} -\dfrac{V_A}{-jX_{C6}} - \dfrac{V_A}{R_1} + \dfrac{V_B}{-jX_{C6}} + \dfrac{E_3}{R_1} - \dfrac{E_1}{R_1} + \underline{J}_2 = 0 \quad \Big|\cdot(-1) \\ V_A\left(\dfrac{1}{-jX_{C6}}\right) - \dfrac{V_B}{jX_{L4}} - \dfrac{V_B}{R_5} - \dfrac{V_B}{-jX_{C6}} + \dfrac{E_3}{jX_{L4}} = 0 \quad \Big|\cdot(-1) \end{cases}$$

$$\begin{cases} V_A(0.05 + j0.05) - V_B(j0.05) = 1 - j + 10 \quad \Big|\cdot 100 \\ -V_A(j0.05) + V_B(0.1) = -j \quad \Big|\cdot 100 \end{cases}$$

$$\begin{cases} V_A\left(\dfrac{1}{-jX_{C6}} + \dfrac{1}{R_1}\right) - V_B\left(\dfrac{1}{-jX_{C6}}\right) = \dfrac{E_3}{R_1} - \dfrac{E_1}{R_1} + \underline{J}_2 \\ -V_A\left(\dfrac{1}{-jX_{C6}}\right) + V_B\left(\dfrac{1}{jX_{L4}} + \dfrac{1}{R_5} + \dfrac{1}{-jX_{C6}}\right) = \dfrac{E_3}{jX_{L4}} \end{cases}$$

$$\begin{cases} V_A(5 + j5) - V_B(j5) = 1100 - j100 \\ -V_A(j5) + V_B(10) = -j100 \end{cases}$$

$$W = \begin{vmatrix} 5+j5 & -j5 \\ -j5 & 10 \end{vmatrix} = 10\cdot(5+j5) - (-j5)\cdot(-j5) = 50 + j50 + 25 = 75 + j50$$

$$W_A = \begin{vmatrix} 1100-j100 & -j5 \\ -j100 & 10 \end{vmatrix} = (1100-j100)\cdot 10 - (-j100)\cdot(-j5) = 11000 - j1000 + 500 = 11500 - j1000$$

$$W_B = \begin{vmatrix} 5+j5 & 1100-j100 \\ -j5 & -j100 \end{vmatrix} = (5+j5)(-j100) - (-j5)\cdot(1100-j100) = 500 - j500 + 500 + j5500 = 1000 + j5000$$

$$\underline{V_A} = \frac{\underline{W_A}}{\underline{W}} = \frac{11500 - j1000}{75 + j50} = \boxed{(100 - j80)\,V}$$

$$\underline{V_B} = \frac{\underline{W_B}}{\underline{W}} = \frac{1000 + j5000}{75 + j50} = \boxed{(40 + j40)\,V}$$

$$\underline{I_1} = \frac{\underline{V_A} - \underline{V_C} + \underline{E_1}}{R_1} = \frac{100 - j80 - 20 + j20}{20} = \frac{80 - j60}{20} = \boxed{(4 - j3)\,A}$$

$$\underline{I_2} = \underline{J_2} = \boxed{10\,A}$$

$$\underline{I_4} = \frac{\underline{V_C} - \underline{V_B}}{jX_{L4}} = \frac{20 - 40 - j40}{j20} = \frac{-20 - j40}{j20} = \boxed{(-2 + j1)\,A}$$

$$\underline{I_3} = \underline{I_4} - \underline{I_1} = -2 + j1 - 4 + j3 = \boxed{(-6 + j4)\,A}$$

$$\underline{I_5} = \frac{\underline{V_B}}{R_5} = \frac{40 + j40}{10} = \boxed{(4 + j4)\,A}$$

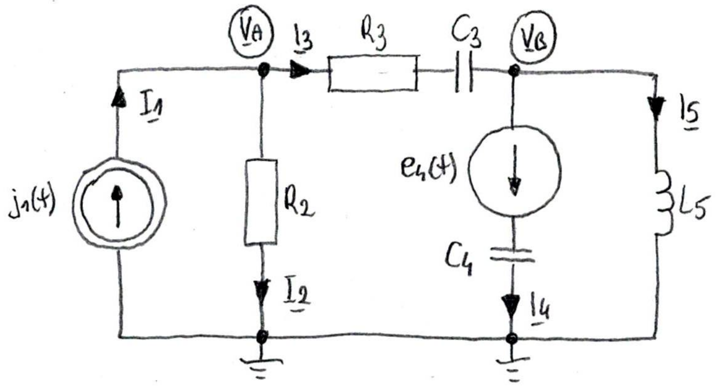
$$\underline{I_6} = \frac{\underline{V_B} - \underline{V_A}}{-jX_{C6}} = \frac{40 + j40 - 100 + j80}{-j20} = \frac{-60 + j120}{-j20} = \boxed{(-6 - j3)\,A}$$

PROBLEM #3

Calculate the currents in all branches of the circuit shown in the figure using the Node-Voltage Method. The parameters are as follows:

$j_1(t) = 10\sqrt{2}\sin(\omega t)\,A$, $e_4(t) = 100\sqrt{2}\sin(\omega t + 90°)\,V$, $\omega = 1000$ rad/s

$R_2 = 10\,\Omega$, $R_3 = 10\,\Omega$, $C_3 = 50\,\mu F$, $C_4 = 50\,\mu F$, $L_5 = 10\,mH$.



$\underline{J_1} = 10\,A \quad \underline{E_4} = j100\,V$

$$X_{C3} = X_{C4} = \frac{1}{\omega C_3} = \frac{1}{1000 \cdot 50 \cdot 10^{-6}} = 20\,\Omega$$

$$X_{L5} = \omega L_5 = 1000 \cdot 0.01 = 10\,\Omega$$

$$\begin{cases} (V_A): & \underline{I_1} - \underline{I_2} - \underline{I_3} = 0 \\ (V_B): & \underline{I_3} - \underline{I_4} - \underline{I_5} = 0 \end{cases}$$

$$\underline{I_1} = \underline{J_1} \qquad \underline{I_2} = \frac{\underline{V_A} - 0}{R_2} \qquad \underline{I_3} = \frac{\underline{V_A} - \underline{V_B}}{R_3 - jX_{C3}} \qquad \underline{I_4} = \frac{\underline{V_B} - 0 + \underline{E_4}}{-jX_{C4}} \qquad \underline{I_5} = \frac{\underline{V_B} - 0}{jX_{L5}}$$

$$\begin{cases} \underline{J}_1 - \dfrac{\underline{V}_A}{R_2} - \dfrac{\underline{V}_A - \underline{V}_B}{R_3 - jX_{C3}} = 0 \\[2ex] \dfrac{\underline{V}_A - \underline{V}_B}{R_3 - jX_{C3}} - \dfrac{\underline{V}_B + \underline{E}_4}{-jX_{C4}} - \dfrac{\underline{V}_B}{jX_{L5}} = 0 \end{cases}$$

$$\begin{cases} -\dfrac{\underline{V}_A}{R_2} - \dfrac{\underline{V}_A}{R_3 - jX_{C3}} + \dfrac{\underline{V}_B}{R_3 - jX_{C3}} = -\underline{J}_1 \quad \Big| \cdot (-1) \\[2ex] \dfrac{\underline{V}_A}{R_3 - jX_{C3}} - \dfrac{\underline{V}_B}{R_3 - jX_{C3}} - \dfrac{\underline{V}_B}{-jX_{C4}} - \dfrac{\underline{V}_B}{jX_{L5}} = \dfrac{\underline{E}_4}{-jX_{C4}} \quad \Big| \cdot (-1) \end{cases}$$

$$\begin{cases} \underline{V}_A \left(\dfrac{1}{R_2} + \dfrac{1}{R_3 - jX_{C3}} \right) - \underline{V}_B \left(\dfrac{1}{R_3 - jX_{C3}} \right) = \underline{J}_1 \\[2ex] -\underline{V}_A \left(\dfrac{1}{R_3 - jX_{C3}} \right) + \underline{V}_B \left(\dfrac{1}{R_3 - jX_{C3}} + \dfrac{1}{-jX_{C4}} + \dfrac{1}{jX_{L5}} \right) = -\dfrac{\underline{E}_4}{-jX_{C4}} \end{cases}$$

$$\begin{cases} \underline{V}_A \left(\dfrac{1}{10} + \dfrac{1}{10 - j20} \right) - \underline{V}_B \left(\dfrac{1}{10 - j20} \right) = 10 \\[2ex] -\underline{V}_A \left(\dfrac{1}{10 - j20} \right) + \underline{V}_B \left(\dfrac{1}{10 - j20} + \dfrac{1}{-j20} + \dfrac{1}{j10} \right) = -\dfrac{j100}{-j20} \end{cases}$$

$$\begin{cases} \underline{V}_A (0.1 + 0.02 + j0.04) - \underline{V}_B (0.02 + j0.04) = 10 \quad \big| \cdot 100 \\ -\underline{V}_A (0.02 + j0.04) + \underline{V}_B (0.02 + j0.04 + j0.05 - j0.1) = 5 \quad \big| \cdot 100 \end{cases}$$

$$\begin{cases} \underline{V}_A (12 + j4) - \underline{V}_B (2 + j4) = 1000 \\ -\underline{V}_A (2 + j4) + \underline{V}_B (2 - j) = 500 \end{cases}$$

$$W = \begin{vmatrix} 12+j4 & -(2+j4) \\ -(2+j4) & 2-j \end{vmatrix} = 40 - j20$$

$$W_A = \begin{vmatrix} 1000 & -(2+j4) \\ 500 & 2-j \end{vmatrix} = 3000 + j1000$$

$$W_B = \begin{vmatrix} 12+j4 & 1000 \\ -(2+j4) & 500 \end{vmatrix} = 8000 + j6000$$

$$\underline{V}_A = \frac{W_A}{W} = \frac{3000 + j1000}{40 - j20} = \boxed{(50 + j50)\,V}$$

$$\underline{V}_B = \frac{W_B}{W} = \frac{8000 + j6000}{40 - j20} = \boxed{(100 + j200)\,V}$$

$$\underline{I}_1 = \underline{J}_1 = \boxed{10A}$$

$$\underline{I}_2 = \frac{\underline{V}_A}{R_2} = \frac{50 + j50}{10} = \boxed{(5 + j5)\,A}$$

$$\underline{I}_3 = \frac{\underline{V}_A - \underline{V}_B}{R_3 - jX_{C3}} = \frac{50 + j50 - 100 - j200}{10 - j20} = \boxed{(5 - j5)\,A}$$

$$\underline{I}_4 = \frac{\underline{V}_B + \underline{E}_4}{-jX_{C4}} = \frac{100 + j200 + j100}{-j20} = \boxed{(-15 + j5)\,A}$$

$$\underline{I}_5 = \frac{\underline{V}_B}{jX_{L5}} = \frac{100 + j200}{j10} = \boxed{(20 - j10)\,A}$$