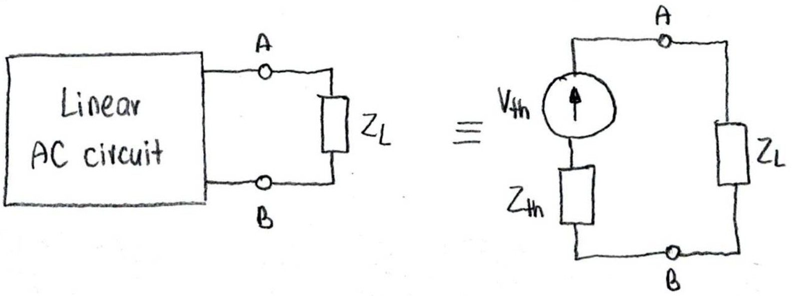
ELECTRICAL CIRCUITS 1 - CLASS NO. 13 (20.01.2025)

THEVENIN'S THEOREM

* any linear AC circuit containing voltage sources, current sources, resistors, inductors and capacitors can be replaced at terminals A-B by an equivalent AC voltage source V_{th} in series with an equivalent impedance Z_{th}

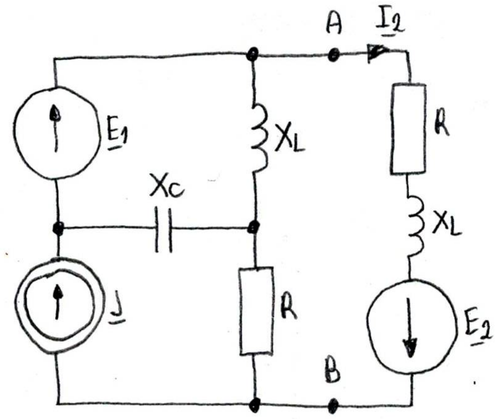


$\underline{V}_{th}$ - the equivalent voltage obtained at terminals A-B of the circuit with terminals A-B open circuited

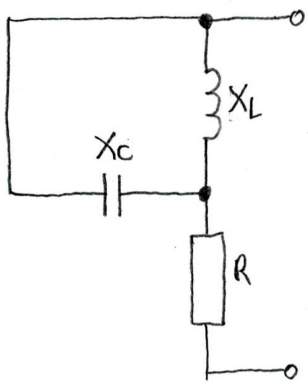
$\underline{Z}_{th}$ - the equivalent impedance between terminals A-B with all sources replaced by their internal impedances (all the voltage sources are replaced by a short, all the current sources are replaced by an open)

PROBLEM #1

Determine the current $\underline{I}_2$ using Thevenin's Theorem. $\underline{E}_1 = 3V$, $\underline{E}_2 = j8V$, $\underline{J} = 2A$, $R = 3\Omega$, $X_L = 4\Omega$, $X_C = 2\Omega$.

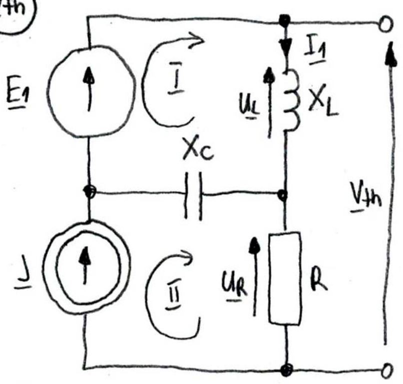


$\underline{Z}_{th}$



$$\underline{Z}_{th} = R + \frac{jX_L \cdot (-jX_C)}{jX_L - jX_C} = 3 + \frac{j4 \cdot (-j2)}{j4 - j2} = 3 + \frac{8}{j2} = \boxed{(3 - j4)\,\Omega}$$

$\underline{V}_{th}$

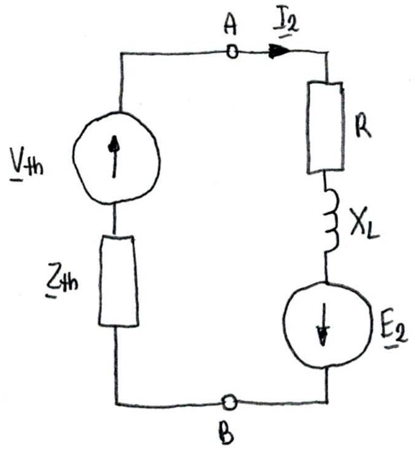


$\underline{I}_{II} = \underline{J}$

$$\underline{I}_I (jX_L - jX_C) - \underline{J}(-jX_C) = \underline{E}_1$$

$$\underline{I}_I = \frac{\underline{E}_1 - jX_C \cdot \underline{J}}{jX_L - jX_C} = \frac{3 - j2 \cdot 2}{j4 - j2} = \frac{3 - j4}{j2} = (-2 - j1.5)A$$

$$\underline{V}_{th} = \underline{U}_R + \underline{U}_L = \underline{J} \cdot R + \underline{I}_I \cdot jX_L = 2 \cdot 3 + (-2 - j1.5) \cdot j4 = \boxed{(12 - j8)V}$$

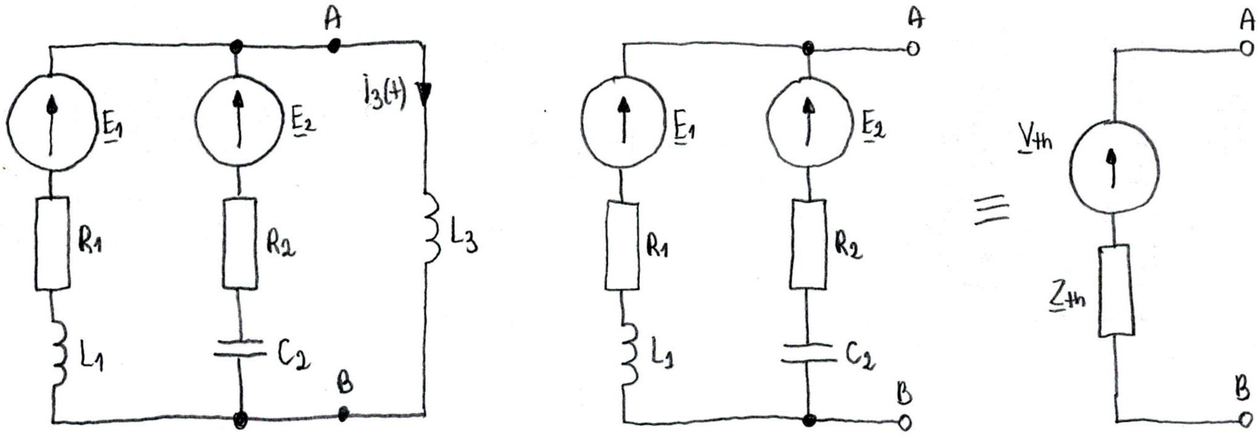


$$\underline{I}_2 = \frac{\underline{V}_{th}+\underline{E}_2}{R+jX_L+\underline{Z}_{th}} = \frac{12-\cancel{j8}+\cancel{j8}}{3+\cancel{j4}+3-\cancel{j4}} = \frac{12}{6} = \boxed{2\,A}$$

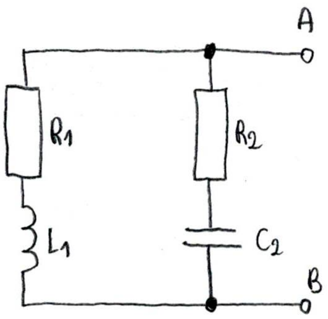
PROBLEM #2

Determine the current $i_3(t)$ using Thevenin's Theorem. $e_1(t) = 200\sin(\omega t+45°)\,V$, $e_2(t) = 100\sqrt{2}\cos(\omega t)\,V$, $R_1 = 20\,\Omega$, $X_{L1} = 80\,\Omega$, $R_2 = 40\,\Omega$, $X_{C2} = 40\,\Omega$, $X_{L3} = 20\,\Omega$.

$e_2(t) = 100\sqrt{2}\sin(\omega t+90°)$



$\underline{Z}_{th}$



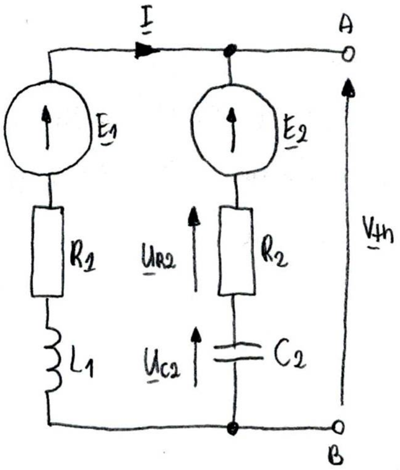
$$\underline{Z}_{th} = \frac{(R_1+jX_{L1})(R_2-jX_{C2})}{R_1+jX_{L1}+R_2-jX_{C2}} = \frac{(20+j80)(40-j40)}{20+j80+40-j40} =$$

$$= \frac{4000+j2400}{60+j40} = \boxed{(64.62-j3.08)\,\Omega}$$

$$\underline{E}_1 = \frac{200}{\sqrt{2}}(\cos 45° + j\sin 45°) = \frac{200}{\cancel{\sqrt{2}}}\left(\frac{\cancel{\sqrt{2}}}{2}+j\frac{\cancel{\sqrt{2}}}{2}\right) = (100+j100)\,V$$

$$\underline{E}_2 = \frac{100\sqrt{2}}{\sqrt{2}}(\cos 90° + j\sin 90°) = 100\,(0+j1) = j100\,V$$

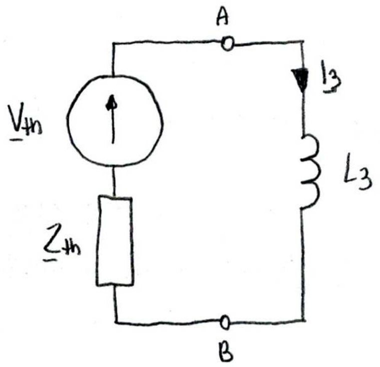
$\underline{V}_{th}$



$$\underline{I} = \frac{\underline{E}_1 - \underline{E}_2}{R_1 + jX_{L1} + R_2 - jX_{C2}} = \frac{100 + j100 - j100}{20 + j80 + 40 - j40} = \frac{100}{60 + j40} = (1.15 - j0.77)\,A$$

$$\underline{V}_{th} = \underline{E}_2 + \underline{U}_{R2} + \underline{U}_{C2} = \underline{E}_2 + R_2 \cdot \underline{I} - jX_{C2} \cdot \underline{I} =$$

$$= j100 + 40(1.15 - j0.77) - j40(1.15 - j0.77) = \boxed{(15.38 + j23.08)\,V}$$



$$\underline{I}_3 = \frac{\underline{V}_{th}}{\underline{Z}_{th} + jX_{L3}} = \frac{15.38 + j23.08}{64.62 - j3.08 + j30} = \frac{15.38 + j23.08}{64.62 + j26.92} =$$

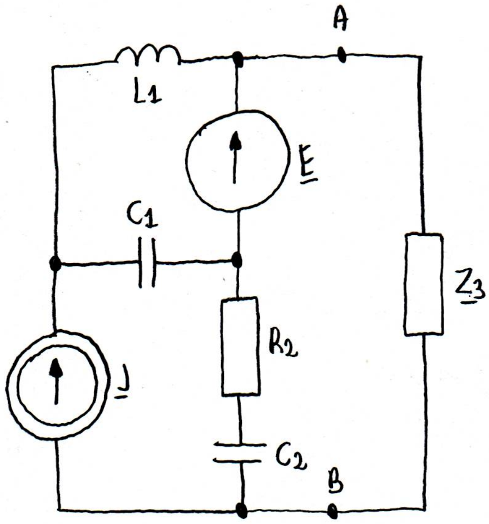
$$= \boxed{(0.31 + j0.28)\,A} = \boxed{0.42\,e^{j41.63^\circ}\,A}$$

$$i_3(t) = 0.42\sqrt{2}\,\sin(\omega t + 41.63^\circ)\,A$$

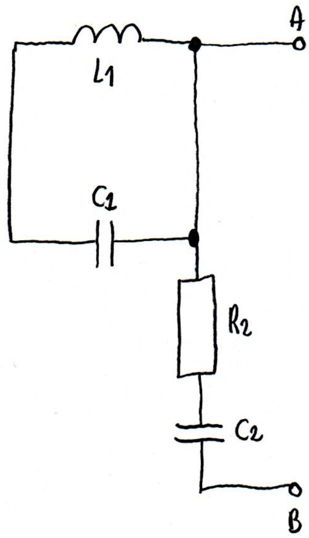
PROBLEM #3

Calculate an active power on $\underline{Z}_3$. Use Thevenin's Theorem.

$\underline{E} = (10 + j10)V$, $\underline{J} = (2 + j2)A$, $X_{L1} = 20\,\Omega$, $X_{C1} = 10\,\Omega$, $R_2 = 10\,\Omega$, $X_{C2} = 10\,\Omega$, $\underline{Z}_3 = (10 + j10)\,\Omega$



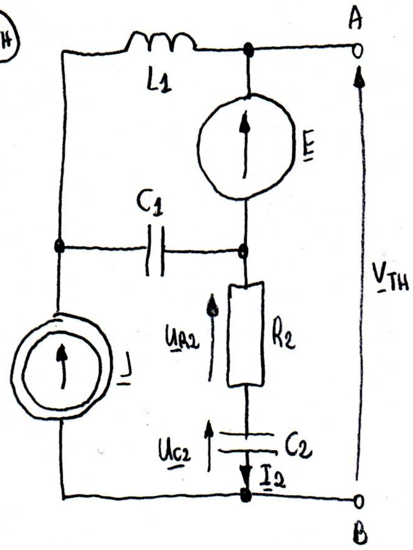
$\underline{Z}_{TH}$



$$\underline{Z}_{TH} = R_2 - jX_{C2} =$$

$$= \boxed{(10 - j10)\,\Omega}$$

$\underline{V}_{TH}$



$\underline{I}_2 = \underline{J}$

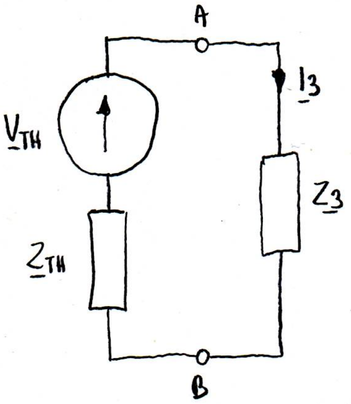
$\underline{V}_{TH} = \underline{E} + \underline{U}_{R2} + \underline{U}_{C2}$

$\underline{V}_{TH} = \underline{E} + (R_2 - jX_{C2}) \cdot \underline{I}_2$

$\underline{V}_{TH} = \underline{E} + (R_2 - jX_{C2}) \underline{J}$

$\underline{V}_{TH} = (10+j10) + (10-j10)(2+j2) = 10+j10+20+j20-j20+20 =$

$= \boxed{(50+j10)\,V}$



$\underline{I}_3 = \dfrac{\underline{V}_{TH}}{\underline{Z}_{TH} + \underline{Z}_3} = \dfrac{50+j10}{10-j10+10+j10} = \dfrac{50+j10}{20} = (2.5+j0.5)\,A$

Method 1

$|\underline{I}_3| = \sqrt{2.5^2 + 0.5^2} = \sqrt{6.25+0.25} = \sqrt{6.5}$

$P = Re\{\underline{Z}_3\} \cdot |\underline{I}_3|^2 = 10 \cdot (\sqrt{6.5})^2 = 10 \cdot 6.5 = \boxed{65\,W}$

Method 2

$\underline{U}_3 = \underline{Z}_3 \cdot \underline{I}_3 = (10+j10) \cdot (2.5+j0.5) = 25+j5+j25-5 = (20+j30)\,V$

$\underline{S}_3 = \underline{U}_3 \cdot \underline{I}_3^* = (20+j30)(2.5-j0.5) = 50-j10+j75+15 = (65+j65)\,VA$

$P_3 = Re\{\underline{S}_3\} = \boxed{65\,W}$