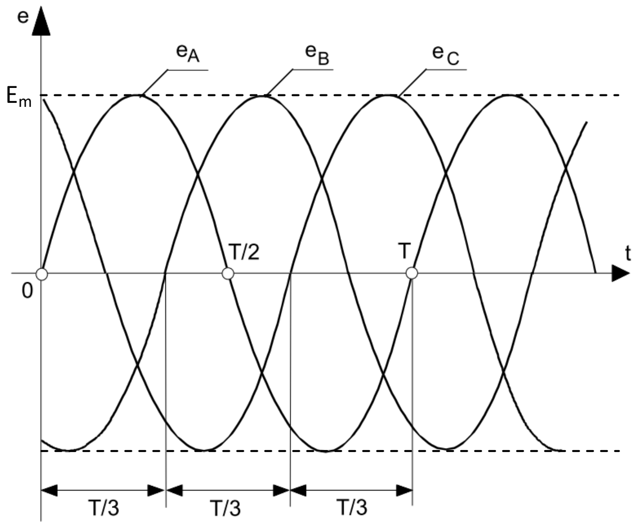
ELECTRICAL CIRCUITS 2 - CLASS 6 (16.04.2024)

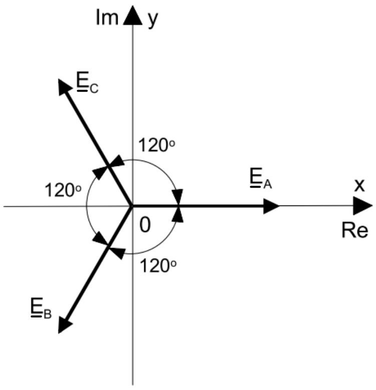
Three-phase circuits

- Generator voltages (positive phase sequence):

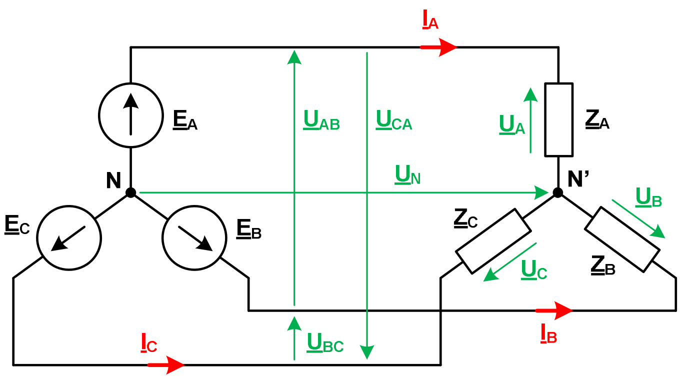
$$e_A(t) = E_m \sin(\omega t) \qquad \underline{E}_A = Ee^{j0°}$$
$$e_B(t) = E_m \sin(\omega t - 120°) \qquad \underline{E}_B = Ee^{-j120°} \qquad \underline{E}_A + \underline{E}_B + \underline{E}_C = 0$$
$$e_C(t) = E_m \sin(\omega t + 120°) \qquad \underline{E}_C = Ee^{j120°}$$

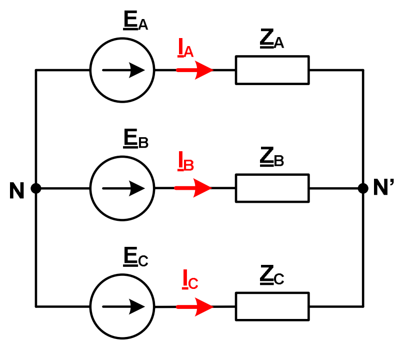
E_m - the peak (maximum) value
$\underline{E}$ - the complex value (phasor)
E - the rms (effective) value





Balanced Wye-Wye (Y-Y) connection, three-wire system





- Symbols in the circuit:

$\underline{E}_A, \underline{E}_B, \underline{E}_C$ - phase voltages of the generator
$\underline{U}_A, \underline{U}_B, \underline{U}_C$ - phase voltages of the load
$\underline{U}_{AB}, \underline{U}_{BC}, \underline{U}_{CA}$ - line voltages (line-to-line)
$\underline{U}_N$ - voltage between N and N' points

$\underline{I}_A, \underline{I}_B, \underline{I}_C$ - the line currents
$\underline{Z}_A, \underline{Z}_B, \underline{Z}_C$ - the load impedances
N - neutral point of the source
N' - neutral point of the load

- Balanced load:

$$\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = \underline{Z}$$

- $\underline{U}_N$ voltage:

$$\underline{U}_N = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} \qquad \text{where:} \qquad \underline{Y}_A = \frac{1}{\underline{Z}_A}, \quad \underline{Y}_B = \frac{1}{\underline{Z}_B}, \quad \underline{Y}_C = \frac{1}{\underline{Z}_C}$$

- Phase voltages of the load:

  $\underline{U}_A = \underline{E}_A, \quad \underline{U}_B = \underline{E}_B, \quad \underline{U}_C = \underline{E}_C$ because in the system with the balanced load: $\underline{U}_N = 0$

- The line currents:

  $$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A}, \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B}, \quad \underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C}$$

  the complex values (phasors): $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$

  the rms (effective) values: $I_A = I_B = I_C$

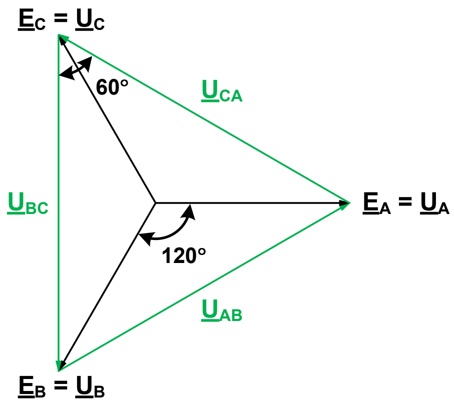
- The line voltages (line-to-line):

  $$\underline{U}_{AB} = \underline{E}_A - \underline{E}_B = \sqrt{3}Ee^{j30^\circ}$$
  $$\underline{U}_{BC} = \underline{E}_B - \underline{E}_C = \sqrt{3}Ee^{-j90^\circ}$$
  $$\underline{U}_{CA} = \underline{E}_C - \underline{E}_A = \sqrt{3}Ee^{j150^\circ}$$

  the complex values (phasors): $\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA} = 0$

  the rms (effective) values: $U_{AB} = U_{BC} = U_{CA}$

- Phasor diagram of voltages:



- Active, reactive and apparent power:

  $$P = U_A \cdot I_A \cdot \cos\varphi_A + U_B \cdot I_B \cdot \cos\varphi_B + U_C \cdot I_C \cdot \cos\varphi_C$$
  $$Q = U_A \cdot I_A \cdot \sin\varphi_A + U_B \cdot I_B \cdot \sin\varphi_B + U_C \cdot I_C \cdot \sin\varphi_C$$
  $$S = U_A \cdot I_A + U_B \cdot I_B + U_C \cdot I_C$$

  Complex power:

  $$\underline{S} = \underline{U}_A \cdot \underline{I}_A^* + \underline{U}_B \cdot \underline{I}_B^* + \underline{U}_C \cdot \underline{I}_C^*$$
  $$\underline{S} = P + jQ$$
  $$P = \mathrm{Re}\{\underline{S}\}, \quad Q = \mathrm{Im}\{\underline{S}\}, \quad S = \|\underline{S}\|$$

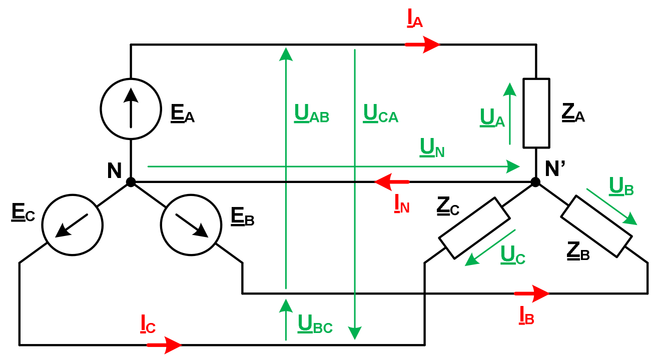
  in the case of balanced load:

  $$U_A = U_B = U_C = U_{ph}, \quad I_A = I_B = I_C = I_{ph}, \quad \cos\varphi_A = \cos\varphi_B = \cos\varphi_C = \cos\varphi_{ph}$$

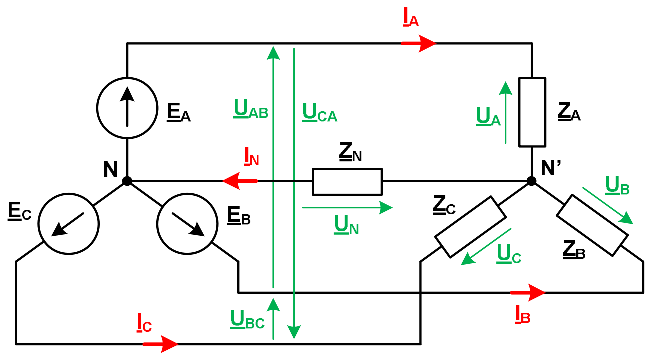
  then:

  $$P = 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos\varphi_{ph}, \quad Q = 3 \cdot U_{ph} \cdot I_{ph} \cdot \sin\varphi_{ph}, \quad S = 3 \cdot U_{ph} \cdot I_{ph}$$

Note: all of the above relationships are also valid for four-wire balanced system with impedance $\underline{Z}_N = 0$ (system on the left) or with impedance $\underline{Z}_N \neq 0$ (system on the right).



$\underline{U}_N = 0, \quad \underline{I}_N = 0$

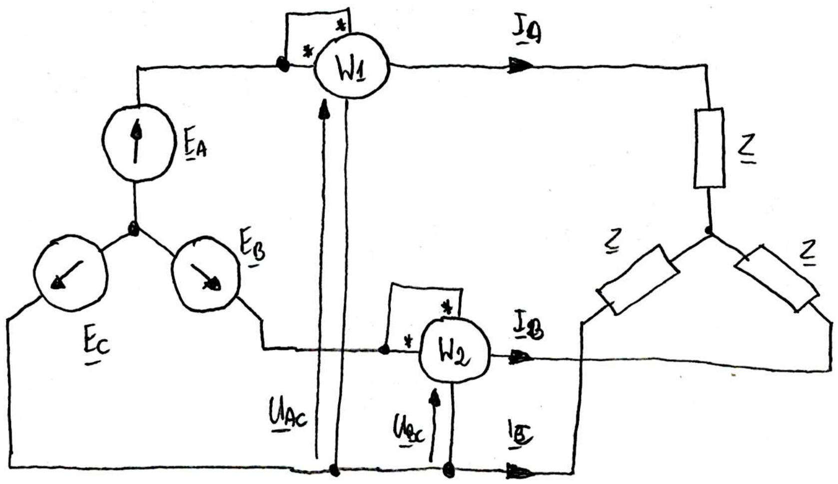


$\underline{U}_N = 0, \quad \underline{I}_N = 0$

ELECTRICAL CIRCUITS 2 - CLASS NO. 6 (16.04.2024)

PROBLEM #1

In a 3-phase balanced Y-Y system, the source voltage is $E_{phase} = 230$ V rms. The impedance per phase is $\underline{Z} = (8+j6)\Omega$. Check whether an overcurrent circuit breaker with a rated current of 10 A is sufficient to protect this circuit. Also, find the active power of the load and the readings of the wattmeters.



$E_{ph} = 230$ V rms

$\underline{E_A} = 230e^{j0°} = 230$ V

$\underline{E_B} = 230e^{-j120°} = (-115 - j199.19)$ V

$\underline{E_C} = 230e^{j120°} = (-115 + j199.19)$ V

$\underline{I_A} = \dfrac{\underline{E_A}}{\underline{Z}} = \dfrac{230}{8+j6} = (18.4 - j13.8)\text{A} = 23e^{-j36.87°}$ A

$\underline{I_B} = \dfrac{\underline{E_B}}{\underline{Z}} = \dfrac{-115-j199.19}{8+j6} = (-21.15 - j9.03)\text{A} = 23e^{-j156.87}$ A

$\underline{I_C} = \dfrac{\underline{E_C}}{\underline{Z}} = \dfrac{-115+j199.19}{8+j6} = (2.75 + j22.83)\text{A} = 23e^{j83.13°}$ A

$|\underline{I_A}| = 23 \text{ A} > 10 \text{ A}$

Method 1

$P = 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos\varphi_{ph}$

$U_{ph} = 230$ V $\quad I_{ph} = 23$ A $\quad \varphi_{ph} = \varphi_u - \varphi_i = 0° - (-36.87°) = 36.87°$

$P_{load} = 3 \cdot 230 \cdot 23 \cdot \cos(36.87°) = \boxed{12\,695.98 \text{ W}}$

Method 2

$\underline{S} = \underline{E_A} \cdot \underline{I_A}^* + \underline{E_B} \cdot \underline{I_B}^* + \underline{E_C} \cdot \underline{I_C}^* = 230 \cdot (18.4 + j13.8) + (-115 - j199.19) \cdot (-21.15 + j9.03) + (-115 + j199.19) \cdot (2.75 - j22.83)$

$= 4232 + j3174 + 4232 + j3174 + 4232 + j3174 = \underbrace{12686}_{P_{load}} + j\,9522$ VA

$P_{load} = \boxed{12\,686 \text{ W}}$

Method 3

$\underline{U_{AC}} = \underline{E_A} - \underline{E_C} = 230 - (-115 + j119.19) = (345 - j199.19)$ V

$\underline{U_{BC}} = \underline{E_B} - \underline{E_C} = -115 - j199.19 - (-115 + j199.19) = -j398.37$ V

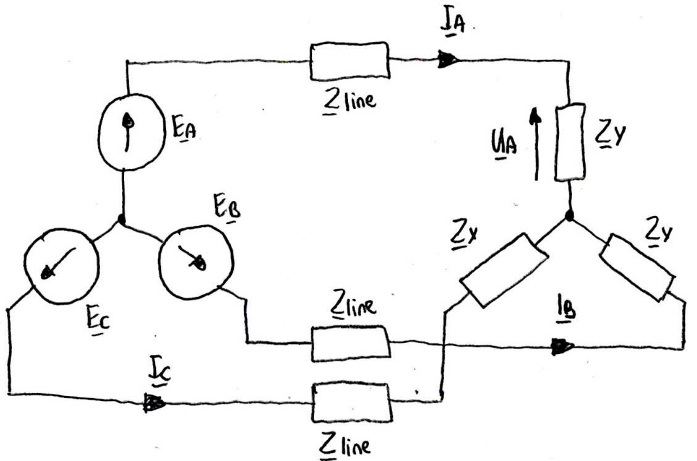
$\underline{S_1} = \underline{U_{AC}} \cdot \underline{I_A}^* = (345 - j199.19) \cdot (18.4 + j13.8) = (\underbrace{9096.76}_{P_{W1}} + j\,1095.98)$ VA $\Rightarrow P_{W1} = 9096.76$ W

$\underline{S_2} = \underline{U_{BC}} \cdot \underline{I_B}^* = (-j398.37) \cdot (-21.15 + j9.03) = (\underbrace{3599.23}_{P_{W2}} + j\,8426.02)$ VA $\Rightarrow P_{W2} = 3599.23$ W

$P_{load} = P_{W1} + P_{W2} \quad P_{W1} + P_{W2} = 9096.76 + 3599.23 = \boxed{12\,695.99 \text{ W}}$

PROBLEM #2

In a 3-phase balanced Y-Y system, the source voltage is $E_{phase} = 230\,V$ rms. The impedance per phase is $\underline{Z}_Y = (9+9j)\,\Omega$ and the line impedance per phase is $(0.5+j0.4)\,\Omega$. What should be the rated current of the overcurrent circuit breakers protecting the circuit? Standard rated currents are: 6A, 10A, 16A, 20A, 25A, 32A, 40A, 50A, 63A, 80A, 125A. Calculate the active power losses in the power line. Also, calculate the percentage voltage drop across the load compared to the rated voltage.



$$\underline{E}_A = 230e^{j0^\circ} = 230\,V$$

$$\underline{E}_B = 230e^{-j120^\circ} = (-115 - j\,199.19)\,V$$

$$\underline{E}_C = 230e^{j120^\circ} = (-115 + j\,199.19)\,V$$

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_Y + \underline{Z}_{line}} = \frac{230}{9+j9+0.5+j0.4} = \frac{230}{9.5+j9.4} =$$

$$= (12.23 - j\,12.10)\,A = 17.21\,e^{-j44.7^\circ}\,A$$

$$|\underline{I}_A| = |\underline{I}_B| = |\underline{I}_C| = \boxed{17.21\,A}$$

The rated current of the overcurrent circuit breaker should be: $\boxed{20\,A}$

The active power losses in the power line:

$$\underline{Z}_{line} = R_{line} + j\,X_{line} = (0.5+j\,0.4)\,\Omega$$

$$P_{losses} = 3 \cdot R_{line} \cdot I_{line}^2 = 3 \cdot 0.5 \cdot 17.21^2 = 3 \cdot 148.09 = \boxed{444.27\,W}$$

The percentage voltage drop:

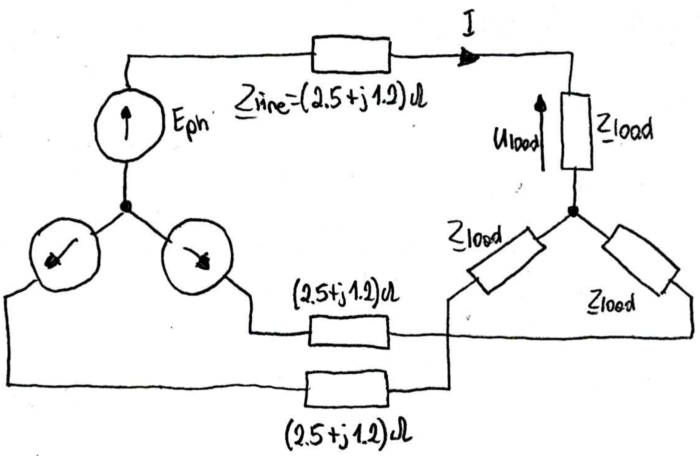
$$\underline{U}_A = \underline{I}_A \cdot \underline{Z}_Y = (12.23 - j\,12.10) \cdot (9+j9) = (219.04 + j\,1.16)\,V = 219.04e^{j0.3^\circ}\,V$$

$$|\underline{U}_A| = 219.04\,V \qquad |\underline{E}_A| = 230\,V$$

$$\Delta U_{\%} = \frac{|\underline{E}_A| - |\underline{U}_A|}{|\underline{E}_A|} \cdot 100\% = \frac{230 - 219.04}{230} \cdot 100\% = \boxed{4.765\%}$$

PROBLEM #3

In the 3-phase balanced Y-Y system, the load voltage is $U_{load} = 400\angle -20^\circ$ V rms, the line impedance is $(2.5+j\,1.2)\,\Omega$, and the source voltage is $E_{phase} = 440$ V rms. Find the load impedance, its power, and the value of supplying current.



$$\underline{I} = \frac{\underline{E}_{ph}}{\underline{Z}_{line} + \underline{Z}_{load}} \qquad \underline{I} = \frac{\underline{U}_{load}}{\underline{Z}_{load}}$$

$$\underline{E}_{ph} \cdot \underline{Z}_{load} = \underline{U}_{load}(\underline{Z}_{line} + \underline{Z}_{load})$$

$$\underline{Z}_{load}(\underline{E}_{ph} - \underline{U}_{load}) = \underline{Z}_{line}\,\underline{U}_{load}$$

$$\boxed{\underline{Z}_{load} = \underline{Z}_{line}\frac{\underline{U}_{load}}{\underline{E}_{ph} - \underline{U}_{load}}}$$

$\underline{U}_{load} = 400e^{-j20°} = (375.88 - j\,136.81)\,V$

$\underline{E}_{ph} = 440e^{j0°} = 440\,V$

$\underline{Z}_{line} = (2.5 + j\,1.2)\,\Omega$

$$\underline{Z}_{load} = (2.5+j\,1.2)\cdot\frac{375.88 - j\,136.81}{440 - 375.88 + j\,136.81} = \boxed{(3.9541 - j\,6.3081)\,\Omega}$$

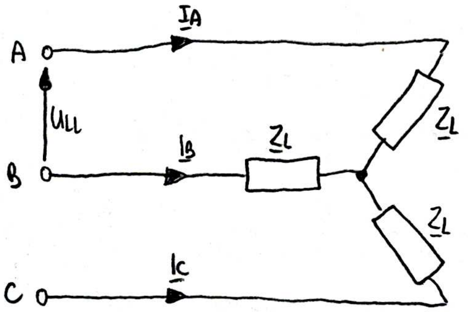
$$\underline{I}_{load} = \frac{\underline{U}_{load}}{\underline{Z}_{load}} = \frac{400e^{-j20°}}{3.7541 - j\,6.3081} = (42.1947 + j\,34.4698)\,A = \boxed{54.48e^{j39.2°}\,A}$$

$$P_{load} = 3\cdot Re\{\underline{S}_{load}\} = 3\cdot Re\{\underline{U}_{load}\cdot \underline{I}^*_{load}\} = 3\cdot Re\{(375.88 - j\,136.81)(42.1947 - j\,34.4698)\} =$$
$$= 3\cdot Re\{11144 - j\,18729\} = 3\cdot 11144 = \boxed{33433\,W}$$

PROBLEM #4

The line-to-line voltage of a balanced 3-phase distribution line is $U_{LL} = 380\,V$ rms.
The load impedance per phase is $\underline{Z}_L = (30+j20)\,\Omega$. Calculate the line currents and the active power of the load for the following configurations of load impedance:
a) a wye-connected system, b) a delta-connected system

a)



$$U_{ph} = \frac{U_{LL}}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 219.3931\,V$$

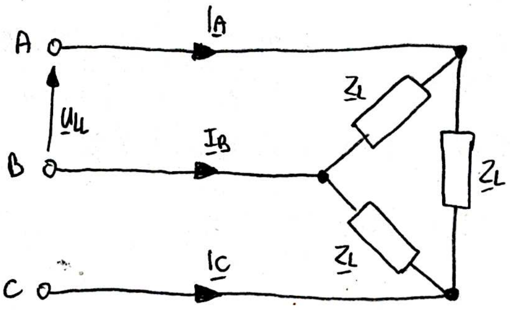
$$\underline{I}_{ph} = \frac{\underline{U}_{ph}}{\underline{Z}_L} = \frac{219.3931}{30+j20} = (5.0629 - j\,3.3753)\,A = 6.08e^{-j33.7°}\,A$$

$$I_A = I_B = I_C = I_{ph} = \boxed{6.08\,A}$$

$$\underline{S} = 3\cdot \underline{U}_{ph}\cdot \underline{I}^*_{ph} = 3\cdot 219.3931\cdot(5.0629 + j\,3.3753) =$$
$$= (3332.3 + j\,2221.5)\,VA$$

$$P = Re\{\underline{S}\} = \boxed{3332.3\,W}$$

b)



$$U_{ph} = U_{LL} = 380\,V$$

$$\underline{I}_{ph} = \frac{\underline{U}_{ph}}{\underline{Z}_L} = \frac{380}{30+j20} = (8.7692 - j\,5.8462)\,A = 10.5393e^{-j33.7}$$

$$\underline{S} = 3\,\underline{U}_{ph}\cdot\underline{I}^*_{ph} = 3\cdot 380\cdot(8.7692 + j\,5.8462) =$$
$$= 9996.9 + j\,6664.6\,VA$$

$$P = Re\{\underline{S}\} = \boxed{9996.9\,W}$$

$$I_A = I_B = I_C = \sqrt{3}\cdot I_{ph} = \boxed{18.25\,A}$$