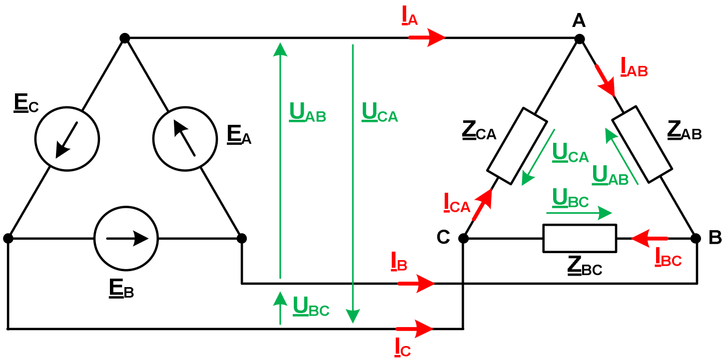
ELECTRICAL CIRCUITS 2 - CLASS 7 (23.04.2024)

Balanced Delta-Delta (Δ-Δ) connection



- Symbols in the circuit:

$\underline{E}_A, \underline{E}_B, \underline{E}_C$ - phase voltages of the generator

$\underline{U}_{AB}, \underline{U}_{BC}, \underline{U}_{CA}$ - phase voltages of the load

$\underline{I}_A, \underline{I}_B, \underline{I}_C$ - the line currents

$\underline{I}_{AB}, \underline{I}_{BC}, \underline{I}_{CA}$ - the phase currents

$\underline{Z}_{AB}, \underline{Z}_{BC}, \underline{Z}_{CA}$ - the load impedances

- Balanced load:

$$\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = \underline{Z}$$

- Phase voltages of the load are equal to phase voltages of the generator:

$$\underline{U}_{AB} = \underline{E}_A, \quad \underline{U}_{BC} = \underline{E}_B, \quad \underline{U}_{CA} = \underline{E}_C$$

the complex values (phasors): $\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA} = 0$

the rms (effective) values: $U_{AB} = U_{BC} = U_{CA}$

- The phase currents:

$$\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{AB}}, \quad \underline{I}_{BC} = \frac{\underline{U}_{BC}}{\underline{Z}_{BC}}, \quad \underline{I}_{CA} = \frac{\underline{U}_{CA}}{\underline{Z}_{CA}}$$

the complex values (phasors): $\underline{I}_{AB} + \underline{I}_{BC} + \underline{I}_{CA} = 0$

the rms (effective) values: $I_{AB} = I_{BC} = I_{CA}$

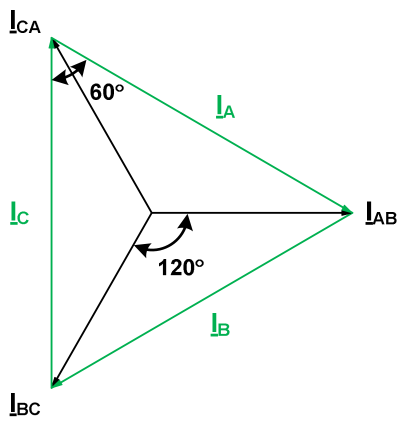
- The line currents:

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA}$$
$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB}$$
$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}$$

the complex values (phasors): $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$

the rms (effective) values: $I_A = I_B = I_C$

- Phasor diagram of currents:



- Active, reactive and apparent power:

$$P = U_{AB} \cdot I_{AB} \cdot \cos\varphi_{AB} + U_{BC} \cdot I_{BC} \cdot \cos\varphi_{BC} + U_{CA} \cdot I_{CA} \cdot \cos\varphi_{CA}$$

$$Q = U_{AB} \cdot I_{AB} \cdot \sin\varphi_{AB} + U_{BC} \cdot I_{BC} \cdot \sin\varphi_{BC} + U_{CA} \cdot I_{CA} \cdot \sin\varphi_{CA}$$

$$S = U_{AB} \cdot I_{AB} + U_{BC} \cdot I_{BC} + U_{CA} \cdot I_{CA}$$

Complex power:

$$\underline{S} = \underline{U}_{AB} \cdot \underline{I}_{AB}^* + \underline{U}_{BC} \cdot \underline{I}_{BC}^* + \underline{U}_{CA} \cdot \underline{I}_{CA}^*$$

$$\underline{S} = P + jQ$$

$$P = \mathrm{Re}\{\underline{S}\}, \quad Q = \mathrm{Im}\{\underline{S}\}, \quad S = \|\underline{S}\|$$

in the case of balanced load:

$$U_{AB} = U_{BC} = U_{CA} = U_{ph}, \quad I_{AB} = I_{BC} = I_{CA} = I_{ph}, \quad \cos\varphi_{AB} = \cos\varphi_{BC} = \cos\varphi_{CA} = \cos\varphi_{ph}$$

then:

$$P = 3 \cdot U_{ph} \cdot I_{ph} \cdot \cos\varphi_{ph}, \quad Q = 3 \cdot U_{ph} \cdot I_{ph} \cdot \sin\varphi_{ph}, \quad S = 3 \cdot U_{ph} \cdot I_{ph}$$

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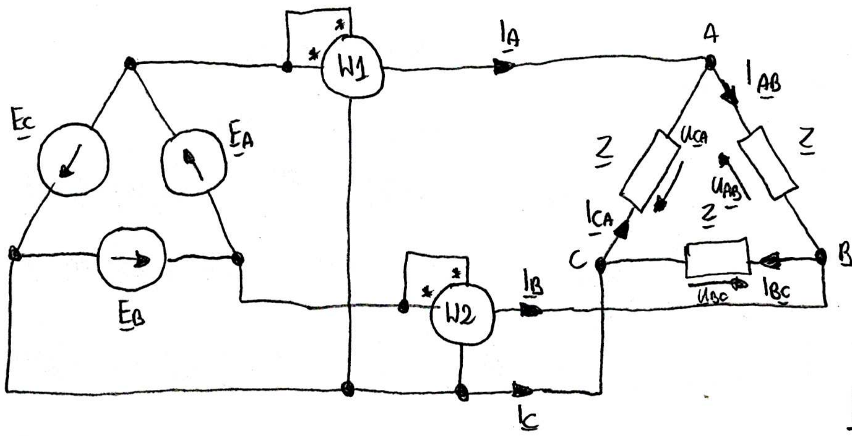
PROBLEM #1

In a 3-phase balanced Δ-Δ system the source voltage $E_{ph} = 230$ V rms. The impedance per phase is $\underline{Z} = (8+j6)\Omega$. Find the line currents, active power of the load and wattmeters readings.



$\underline{E}_A = 230e^{j0°}$ V $= 230$ V

$\underline{E}_B = 230e^{-j120°}$ V $= -115 - j199.19$ V

$\underline{E}_C = 230e^{j120°}$ V $= -115 + j199.19$ V

$\underline{U}_{AB} = \underline{E}_A$

$\underline{U}_{BC} = \underline{E}_B$

$\underline{U}_{CA} = \underline{E}_C$

<u>Note</u>: for $\underline{I}_B$ current, the calculated angle is positive. After changing the angle to negative ($173.13° - 360° = -186.87°$), we can see that the angle between the phase and line current is 30°.

$$\underline{I}_{AB} = \frac{\underline{E}_A}{\underline{Z}} = \frac{230}{8+j6} = (18.4 - j13.8)\,A = 23e^{-j36.87°}\,A$$

$$\underline{I}_{BC} = \frac{\underline{E}_B}{\underline{Z}} = \frac{-115 - j199.19}{8+j6} = (-21.15 - j9.03)\,A = 23e^{-j156.87°}\,A$$

$$\underline{I}_{CA} = \frac{\underline{E}_C}{\underline{Z}} = \frac{-115 + j199.19}{8+j6} = (2.75 + j22.83)\,A = 23e^{j83.13}\,A$$

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA} = 18.4 - j13.8 - 2.75 - j22.83 = (15.65 - j36.63)A = 39.84e^{-j66.87°}A$$

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB} = -21.15 - j9.03 - 18.4 + j13.8 = (-39.55 + j4.77)A = 39.84e^{j173.13°}A = 39.84e^{-j186.87°}A$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC} = 2.75 + j22.83 + 21.15 + j9.03 = (23.9 + j31.87)A = 39.84e^{j53.13°}A = 39.84e^{-j306.87°}A$$

$$\underline{S}_1 = (-\underline{E}_C)\cdot \underline{I}_A^* = (115 - j199.19)\cdot(15.65 + j36.63) = (\underbrace{9096.76}_{P_{W1}} + j1095.88)\,VA \rightarrow P_{W1} = 9096.76\,W$$

$$\underline{S}_2 = \underline{E}_B \cdot \underline{I}_B^* = (-115 - j199.19)(-39.55 - j4.77) = (\underbrace{3599.23}_{P_{W2}} + j8426.02)\,VA \rightarrow P_{W2} = 3599.23\,W$$

$$\underline{S} = \underline{E}_A\cdot\underline{I}_{AB}^* + \underline{E}_B\cdot\underline{I}_{BC}^* + \underline{E}_C\cdot\underline{I}_{CA}^* = 230\cdot(18.4 - j13.8) + (-115 - j199.19)(-21.15 - j9.03) + (-115 + j199.19)(2.75 + j22.83)$$

$P_{load} = P_{W1} + P_{W2}$

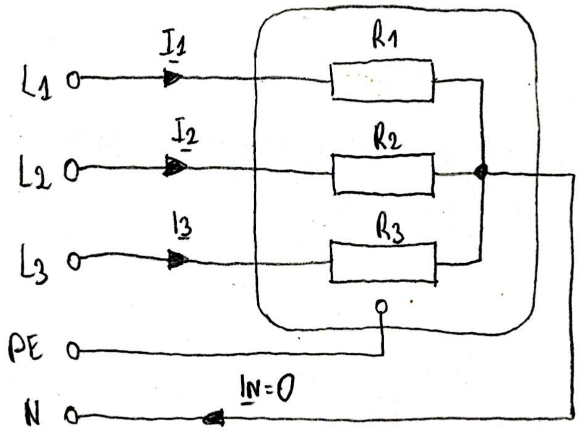
$$= 4232 + j3174 + 4232 + j3174 + 4232 + j3174 = (\underbrace{12696}_{P_{load}} + j9522)\,VA$$

$P_{load} = \boxed{12696\,W}$ $\quad P_{W1} + P_{W2} = 9096.76 + 3599.23 = \boxed{12695.99\,W}$

PROBLEM #2

The three-phase electric heater consists of three heating coils Y-connected. The nominal power of the heater is $P_n = 3$ kW, and the nominal voltage $U_n = 230$ V rms. The heater has been damaged. After its repair the length of the first coil decreased by 5% and the length of the second coil by 10%.

a) calculate the line currents before repairing the heater,



$$P = \frac{P_n}{3} = \frac{3000}{3} = 1000\ \text{W}$$

$$R_1 = R_2 = R_3 = R \qquad R = \frac{U_n^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega$$

$$\underline{E}_1 = 230 e^{j0°} = 230\ \text{V}$$

$$\underline{E}_2 = 230 e^{-j120°} = (-115 - j\,199.19)\ \text{V}$$

$$\underline{E}_3 = 230 e^{+j120°} = (-115 + j\,199.19)\ \text{V}$$

$$\underline{I}_1 = \frac{\underline{E}_1}{R} = \frac{230}{52.9} = 4.3478 = 4.3478 e^{j0°}\ \text{A}$$

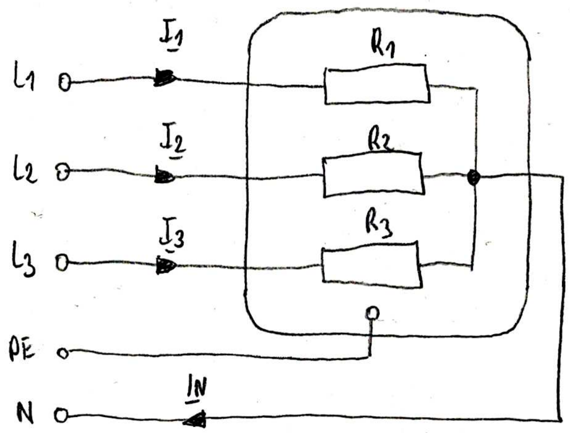
$$\underline{I}_2 = \frac{\underline{E}_2}{R} = \frac{-115 - j\,199.19}{52.9} = (-2.1739 - j\,3.7653) = 4.3478 e^{-j120°}\ \text{A}$$

$$\underline{I}_3 = \frac{\underline{E}_3}{R} = \frac{-115 + j\,199.19}{52.9} = (-2.1739 + j\,3.7653) = 4.3478 e^{+j120°}\ \text{A}$$

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$I_1 = I_2 = I_3$$

b) calculate line currents, the current in the neutral line and the power of the repaired heater



$$R_1 = 0.95 R = 0.95 \cdot 52.9 = 50.255\ \Omega$$

$$R_2 = 0.9 R = 0.9 \cdot 52.9 = 47.61\ \Omega$$

$$R_3 = R = 52.9\ \Omega$$

$$\underline{I}_1 = \frac{\underline{E}_1}{R_1}$$

$$\underline{I}_2 = \frac{\underline{E}_2}{R_2}$$

$$\underline{I}_3 = \frac{\underline{E}_3}{R_3}$$

$$\underline{I}_N = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

$$P_n = \frac{U_n^2}{R_1} + \frac{U_n^2}{R_2} + \frac{U_n^2}{R_3}$$

$$P_n = \text{Re}\{\underline{S}\} = \text{Re}\{\underline{E}_1 \cdot \underline{I}_1^* + \underline{E}_2 \cdot \underline{I}_2^* + \underline{E}_3 \cdot \underline{I}_3^*\}$$