ELECTRICAL CIRCUITS 2 - CLASS 8 (30.04.2024)

Unbalanced Wye-Wye (Y-Y) connection

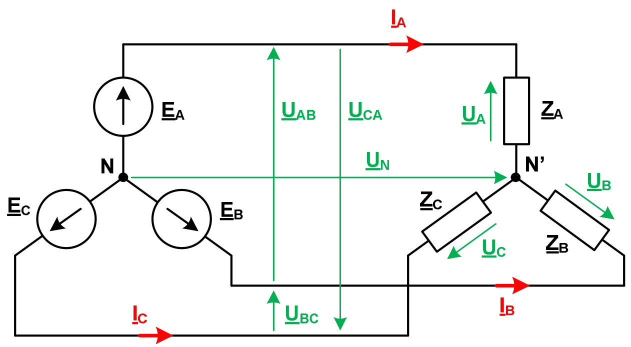
- Generator voltages (positive phase sequence)

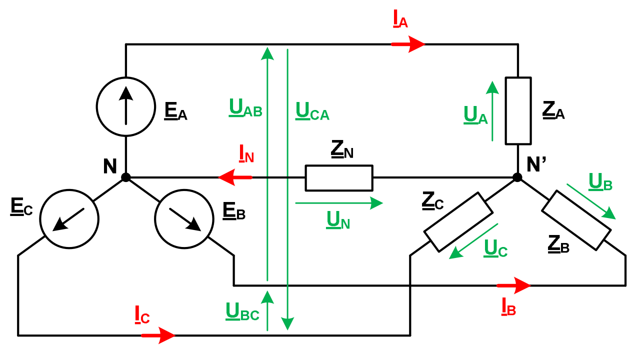
$$e_A(t) = E_m \sin(\omega t) \qquad \underline{E}_A = Ee^{j0^\circ}$$
$$e_B(t) = E_m \sin(\omega t - 120^\circ) \qquad \underline{E}_B = Ee^{-j120^\circ} \qquad \underline{E}_A + \underline{E}_B + \underline{E}_C = 0$$
$$e_C(t) = E_m \sin(\omega t + 120^\circ) \qquad \underline{E}_C = Ee^{j120^\circ}$$

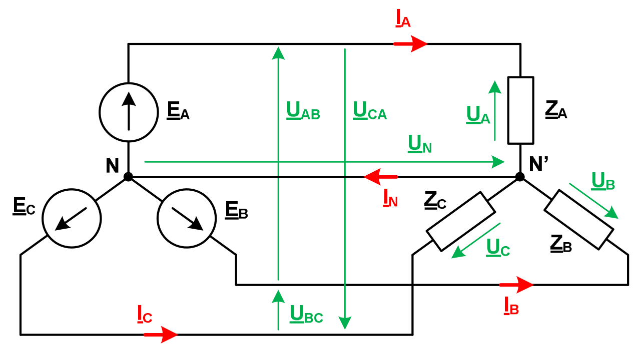
E_m - the peak (maximum) value

$\underline{E}$ - the complex value (phasor)

E - the rms (effective) value







- Unbalanced load: $\underline{Z}_A \neq \underline{Z}_B \neq \underline{Z}_C$

- U_N voltage:

$$\underline{U}_N = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} \neq 0$$

where:

$$\underline{Y}_A = \frac{1}{\underline{Z}_A}, \quad \underline{Y}_B = \frac{1}{\underline{Z}_B}, \quad \underline{Y}_C = \frac{1}{\underline{Z}_C}$$

- U_N voltage:

$$\underline{U}_N = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_N} \neq 0$$

where:

$$\underline{Y}_A = \frac{1}{\underline{Z}_A}, \quad \underline{Y}_B = \frac{1}{\underline{Z}_B}, \quad \underline{Y}_C = \frac{1}{\underline{Z}_C}, \quad \underline{Y}_N = \frac{1}{\underline{Z}_N}$$

- U_N voltage:

$$\underline{U}_N = 0$$

- Phase voltages of the load:

$$\underline{U}_A = \underline{E}_A - \underline{U}_N, \quad \underline{U}_B = \underline{E}_B - \underline{U}_N, \quad \underline{U}_C = \underline{E}_C - \underline{U}_N$$

- Phase voltages of the load:

$$\underline{U}_A = \underline{E}_A - \underline{U}_N, \quad \underline{U}_B = \underline{E}_B - \underline{U}_N, \quad \underline{U}_C = \underline{E}_C - \underline{U}_N$$

- Phase voltages of the load:

$$\underline{U}_A = \underline{E}_A, \quad \underline{U}_B = \underline{E}_B, \quad \underline{U}_C = \underline{E}_C$$

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- The line currents:

$$\underline{I}_A=\frac{\underline{U}_A}{\underline{Z}_A},\quad \underline{I}_B=\frac{\underline{U}_B}{\underline{Z}_B},\quad \underline{I}_C=\frac{\underline{U}_C}{\underline{Z}_C}$$

$$\underline{I}_A+\underline{I}_B+\underline{I}_C=0$$

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- The line currents:

$$\underline{I}_A=\frac{\underline{U}_A}{\underline{Z}_A},\quad \underline{I}_B=\frac{\underline{U}_B}{\underline{Z}_B},\quad \underline{I}_C=\frac{\underline{U}_C}{\underline{Z}_C}$$

$$\underline{I}_A+\underline{I}_B+\underline{I}_C=\underline{I}_N,\quad \underline{I}_N=\frac{\underline{U}_N}{\underline{Z}_N}$$

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- The line currents:

$$\underline{I}_A=\frac{\underline{U}_A}{\underline{Z}_A},\quad \underline{I}_B=\frac{\underline{U}_B}{\underline{Z}_B},\quad \underline{I}_C=\frac{\underline{U}_C}{\underline{Z}_C}$$

$$\underline{I}_A+\underline{I}_B+\underline{I}_C=\underline{I}_N$$

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- The line voltages (line-to-line):

$$\underline{U}_{AB}=\underline{E}_A-\underline{E}_B=\sqrt{3}Ee^{j30^\circ}$$
$$\underline{U}_{BC}=\underline{E}_B-\underline{E}_C=\sqrt{3}Ee^{-j90^\circ}$$
$$\underline{U}_{CA}=\underline{E}_C-\underline{E}_A=\sqrt{3}Ee^{j150^\circ}$$

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- Active, reactive and apparent power:

$$P=U_A\cdot I_A\cdot\cos\varphi_A+U_B\cdot I_B\cdot\cos\varphi_B+U_C\cdot I_C\cdot\cos\varphi_C$$
$$Q=U_A\cdot I_A\cdot\sin\varphi_A+U_B\cdot I_B\cdot\sin\varphi_B+U_C\cdot I_C\cdot\sin\varphi_C$$
$$S=U_A\cdot I_A+U_B\cdot I_B+U_C\cdot I_C$$

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- Complex power: $\underline{S}=\underline{U}_A\cdot\underline{I}_A^*+\underline{U}_B\cdot\underline{I}_B^*+\underline{U}_C\cdot\underline{I}_C^*$ $\quad\underline{S}=P+jQ,\quad P=\mathrm{Re}\{\underline{S}\},\quad Q=\mathrm{Im}\{\underline{S}\}$

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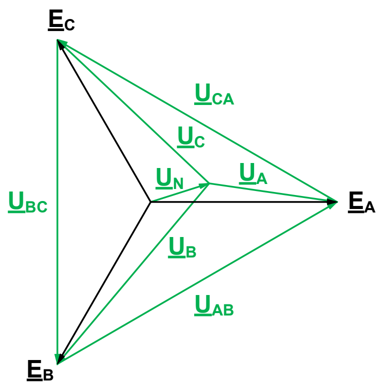
- Phasor diagram of voltages:



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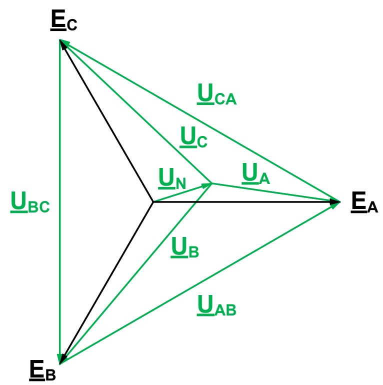

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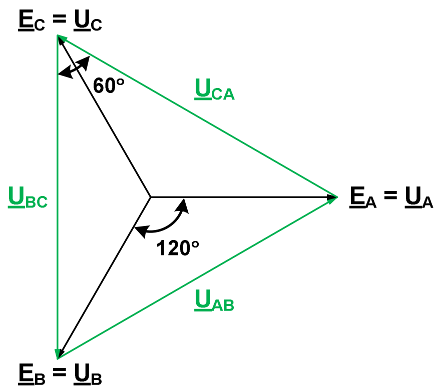

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ELECTRICAL CIRCUITS 2 - CLASS NO. 8 (30.04.2024)

POWER IN AC CIRCUITS

* **P - ACTIVE POWER** (TRUE POWER, REAL POWER) unit: Watt [W]

$P = U_{rms} \cdot I_{rms} \cdot \cos\varphi$, where $\varphi = \psi_u - \psi_i$ (angle between voltage and current)

$U_{rms} = \frac{U_m}{\sqrt{2}}$ $I_{rms} = \frac{I_m}{\sqrt{2}}$ $\rightarrow$ $P = \frac{U_m \cdot I_m}{2} \cdot \cos\varphi$ $P = R \cdot I_{rms}^2 = \frac{U_{rms}^2}{R}$

* **Q - REACTIVE POWER** unit: voltampere (reactive) [var]

$Q = U_{rms} \cdot I_{rms} \cdot \sin\varphi$ $\varphi = \psi_u - \psi_i$ (angle between voltage and current)

$Q = \frac{U_m \cdot I_m}{2} \sin\varphi$ $Q = X \cdot I_{rms}^2 = \frac{U_{rms}^2}{X}$

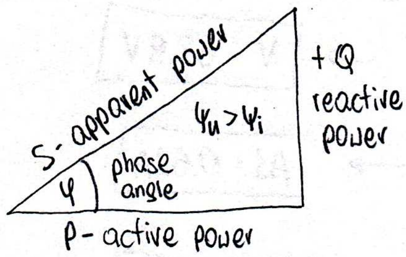
* **S - APPARENT POWER** unit: voltampere [VA]

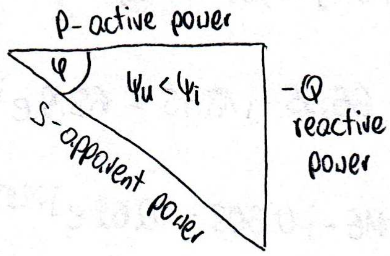
$S = U_{rms} \cdot I_{rms}$ $S = I_{rms}^2 \cdot Z = \frac{U_{rms}^2}{Z}$

* **$\underline{S}$ - PHASOR POWER (COMPLEX POWER)**

$\underline{S} = \underline{U} \cdot \underline{I}^*$ $\underline{S} = P + jQ$ $P = \mathrm{Re}[\underline{S}]$ $Q = \mathrm{Im}[\underline{S}]$

* **THE POWER TRIANGLE**



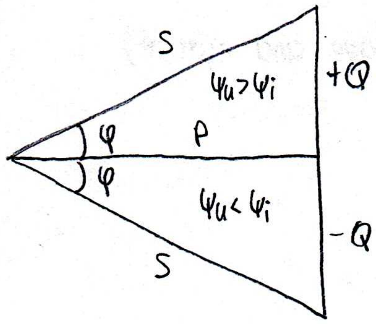


$S = \sqrt{P^2 + Q^2}$

* **THE POWER FACTOR (pf)**

$(pf) = \frac{P}{S} = \cos\varphi$

* the ratio between the active power and the apparent power of a load is called the power factor of the load



lagging power factor

leading power factor

* inductive loads always have a lagging power factor

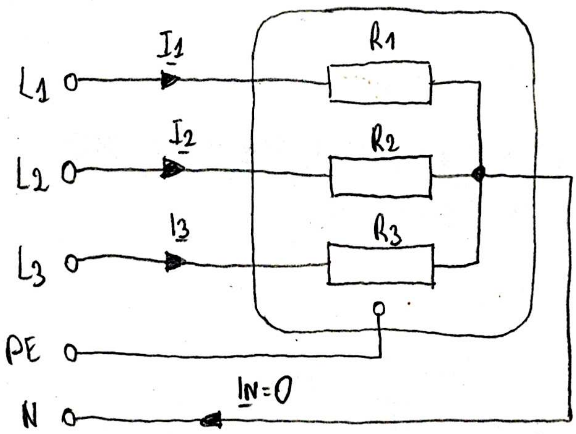
* capacitive loads always have a leading power factor

ELECTRICAL CIRCUITS 2 - CLASS NO. 8 (30.04.2024)

PROBLEM #2, CLASS 7

The three-phase electric heater consists of three heating coils Y-connected. The nominal power of the heater is $P_n = 3$ kW, and the nominal voltage $U_n = 230$ V rms. The heater has been damaged. After its repair the length of the first coil decreased by 5% and the length of the second coil by 10%.

a) calculate the line currents before repairing the heater,



$$P = \frac{P_n}{3} = \frac{3000}{3} = 1000 \text{ W}$$

$$R_1 = R_2 = R_3 = R \qquad R = \frac{U_n^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega$$

$$\underline{E}_1 = 230e^{j0^\circ} = 230 \text{ V}$$

$$\underline{E}_2 = 230e^{-j120^\circ} = (-115 - j\,199.19) \text{ V}$$

$$\underline{E}_3 = 230e^{+j120^\circ} = (-115 + j\,199.19) \text{ V}$$

$$\underline{I}_1 = \frac{\underline{E}_1}{R} = \frac{230}{52.9} = 4.3478 = 4.3478e^{j0^\circ} \text{ A}$$

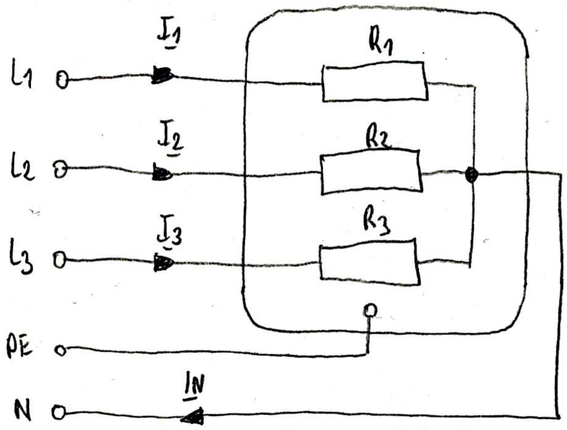
$$\underline{I}_2 = \frac{\underline{E}_2}{R} = \frac{-115 - j\,199.19}{52.9} = (-2.1739 - j\,3.7653) = 4.3478e^{-j120^\circ} \text{ A}$$

$$\underline{I}_3 = \frac{\underline{E}_3}{R} = \frac{-115 + j\,199.19}{52.9} = (-2.1739 + j\,3.7653) = 4.3478e^{+j120^\circ} \text{ A}$$

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$I_1 = I_2 = I_3$$

b) calculate line currents, the current in the neutral line and the power of the repaired heater



$$R_1 = 0.95R = 0.95 \cdot 52.9 = 50.255\ \Omega$$

$$R_2 = 0.9R = 0.9 \cdot 52.9 = 47.61\ \Omega$$

$$R_3 = R = 52.9\ \Omega$$

$$\underline{I}_1 = \frac{\underline{E}_1}{R_1} = \frac{230}{50.255} = 4.5767 = 4.5767e^{j0^\circ} \text{ A}$$

$$\underline{I}_2 = \frac{\underline{E}_2}{R_2} = \frac{(-115 - j\,199.19)}{47.61} = (-2.4155 - j\,4.1837) = 4.8309e^{-j120^\circ} \text{ A}$$

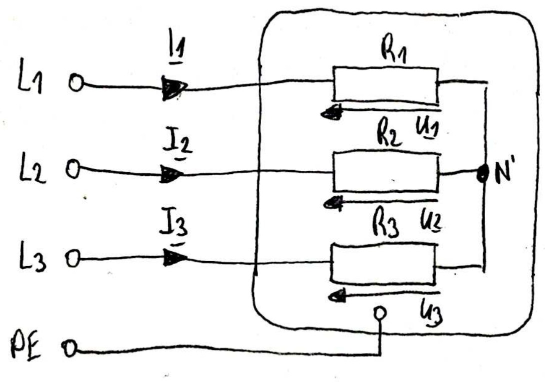
$$\underline{I}_3 = \frac{\underline{E}_3}{R_3} = \frac{(-115 + j\,199.19)}{52.9} = (-2.1739 + j\,3.7653) = 4.3478e^{+j120^\circ} \text{ A}$$

$$\underline{I}_N = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 4.5767 - 2.4155 - j\,4.1837 - 2.1739 + j\,3.7653 = (-0.0127 - j\,0.4184) = 0.4186e^{-j91.74^\circ} \text{ A}$$

$$P_n = \frac{U_n^2}{R_1} + \frac{U_n^2}{R_2} + \frac{U_n^2}{R_3} = \frac{230^2}{50.255} + \frac{230^2}{47.61} + \frac{230^2}{52.9} = 3163.7 \text{ W}$$

$$P_n = \mathrm{Re}\{\underline{S}\} = \mathrm{Re}\{\underline{E}_1 \cdot \underline{I}_1^* + \underline{E}_2 \cdot \underline{I}_2^* + \underline{E}_3 \cdot \underline{I}_3^*\} = \mathrm{Re}\{230 \cdot (4.5767) + (-115 - j\,199.19)(-2.4155 + j\,4.1837) +$$
$$+ (-115 + j\,199.19)(-2.1739 - j\,3.7653)\} = 3163.7 \text{ W}$$

c) Calculate line currents and the power of the repaired heater, when the neutral line is not connected



$$\underline{Y}_1 = \frac{1}{\underline{Z}_1} = \frac{1}{R_1} = \frac{1}{50.255} = 0.0199\ \Omega^{-1}$$

$$\underline{Y}_2 = \frac{1}{\underline{Z}_2} = \frac{1}{R_2} = \frac{1}{47.61} = 0.021\ \Omega^{-1}$$

$$\underline{Y}_3 = \frac{1}{\underline{Z}_3} = \frac{1}{R_3} = \frac{1}{52.9} = 0.0189\ \Omega^{-1}$$

$$\underline{U}_{NN'} = \frac{\underline{E}_1 \cdot \underline{Y}_1 + \underline{E}_2 \cdot \underline{Y}_2 + \underline{E}_3 \cdot \underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} = \frac{230 \cdot 0.0199 + (-115 - j199.19) \cdot 0.021 + (-115 + j199.19) \cdot 0.0189}{0.0199 + 0.021 + 0.0189} =$$

$$= (-0.2126 - j6.9954)\,\mathrm{V}$$

$$\underline{I}_1 = \frac{\underline{U}_1}{R_1} = \frac{\underline{E}_1 - \underline{U}_{NN'}}{R_1} = \frac{230 + 0.2126 + j6.9954}{50.255} = 4.5809 + j0.1392 = 4.583 e^{+j1.74^\circ}\,\mathrm{A}$$

$$\underline{I}_2 = \frac{\underline{U}_2}{R_2} = \frac{\underline{E}_2 - \underline{U}_{NN'}}{R_2} = \frac{(-115 - j199.19) + 0.2126 + j6.9954}{47.61} = -2.411 - j4.0368 = 4.702 e^{-j120.85^\circ}\,\mathrm{A}$$

$$\underline{I}_3 = \frac{\underline{U}_3}{R_3} = \frac{\underline{E}_3 - \underline{U}_{NN'}}{R_3} = \frac{(-115 + j199.19) + 0.2126 + j6.9954}{52.9} = -2.1699 + j3.8976 = 4.4609 e^{+j119.11^\circ}\,\mathrm{A}$$

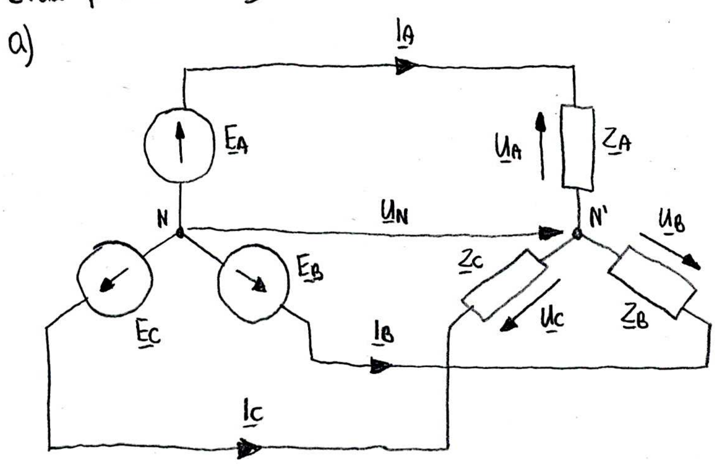
PROBLEM #1

In a 3-phase unbalanced Y-Y system, the source voltage is $E_{ph} = 230$ V rms. The load impedances are: $\underline{Z}_A = (6+j8)\,\Omega$, $\underline{Z}_B = (8+j6)\,\Omega$, $\underline{Z}_C = 20\,\Omega$. What should be the rated current of the overcurrent circuit breakers protecting this circuit? Standard rated currents are: 6 A, 10 A, 16 A, 20 A, 25 A, 32 A, 40 A, 50 A, 63 A, 80 A, 125 A. What is the cost of active energy used by this load during one week (8 h per day, 5 days, 0.299 €/kWh)? Consider two cases:

a) without a neutral wire, b) with a neutral wire.

Draw phasor diagrams of currents and voltages for both cases.

a)



$$\underline{E}_A = 230 e^{j0^\circ} = 230\ \mathrm{V}$$

$$\underline{E}_B = 230 e^{-j120^\circ} = (-115 - j199.19)\ \mathrm{V}$$

$$\underline{E}_C = 230 e^{+j120^\circ} = (-115 + j199.19)\ \mathrm{V}$$

$$\underline{Y}_A = 1/\underline{Z}_A = 1/(6+j8) = (0.06 - j0.08)\ \mathrm{S}$$

$$\underline{Y}_B = 1/\underline{Z}_B = 1/(8+j6) = (0.08 - j0.06)\ \mathrm{S}$$

$$\underline{Y}_C = 1/\underline{Z}_C = 1/20 = 0.05\ \mathrm{S}$$

$$\underline{U}_N = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} = \frac{(230)\cdot(0.06-j0.08) + (-115-j199.19)\cdot(0.08-j0.06) + (-115+j199.19)\cdot(0.05)}{0.06-j0.08+0.08-j0.06+0.05} =$$

$$= (-0.77 - j92.54) = 92.54\,e^{-j90.47^\circ}\ \mathrm{V}$$

$$\underline{U}_A = \underline{E}_A - \underline{U}_N = 230 - (-0.77 - j92.54) = (230.77 + j92.54)\ \mathrm{V} = 248.63\,e^{j21.85^\circ}\ \mathrm{V}$$

$$\underline{U}_B = \underline{E}_B - \underline{U}_N = (-115 - j199.19) - (-0.77 - j92.54) = (-114.23 - j106.65)\ \mathrm{V} = 156.28\,e^{-j136.97^\circ}\ \mathrm{V}$$

$$\underline{U}_C = \underline{E}_C - \underline{U}_N = (-115 + j199.19) - (-0.77 - j92.54) = (-114.23 + j291.73)\ \mathrm{V} = 313.30\,e^{j111.38^\circ}\ \mathrm{V}$$

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_A} = \frac{230.77 + j92.54}{6+j8} = (21.25 - j12.91)\ \mathrm{A} = 24.86\,e^{-j31.28^\circ}\ \mathrm{A}$$

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_B} = \frac{-114.23 - j106.65}{8+j6} = (-15.54 - j1.68)\ \mathrm{A} = 15.63\,e^{-j173.84^\circ}\ \mathrm{A}$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \frac{-114.23 + j291.73}{20} = (-5.71 + j14.58)\ \mathrm{A} = 15.66\,e^{j111.38^\circ}\ \mathrm{A}$$

The rated current of the overcurrent circuit breaker:

$$\boxed{I_n = 25\ \mathrm{A}}$$

$$P_{TOTAL} = \mathrm{Re}\{\underline{Z}_A\} I_A^2 + \mathrm{Re}\{\underline{Z}_B\} \cdot I_B^2 + \mathrm{Re}\{\underline{Z}_C\} \cdot I_C^2 =$$

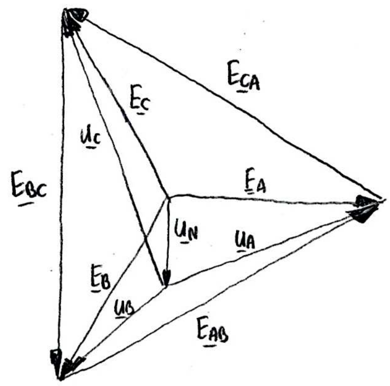
$$= 6 \cdot 24.86^2 + 8 \cdot 15.63^2 + 20 \cdot 15.66^2 =$$

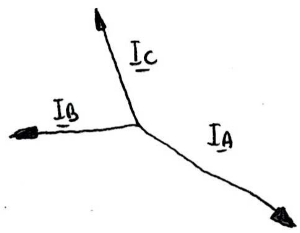
$$= 10\,570.5\ \mathrm{W}$$

$$W = P_{TOTAL} \cdot \mathrm{time} = 10570.5 \cdot 8 \cdot 5 =$$

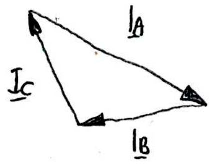
$$= 422\,820\ \mathrm{Wh} = 422.82\ \mathrm{kWh}$$

$$\mathrm{cost} = 422.82 \cdot 0.289 = \boxed{122.2\ \mathrm{Euro}}$$





$$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$$



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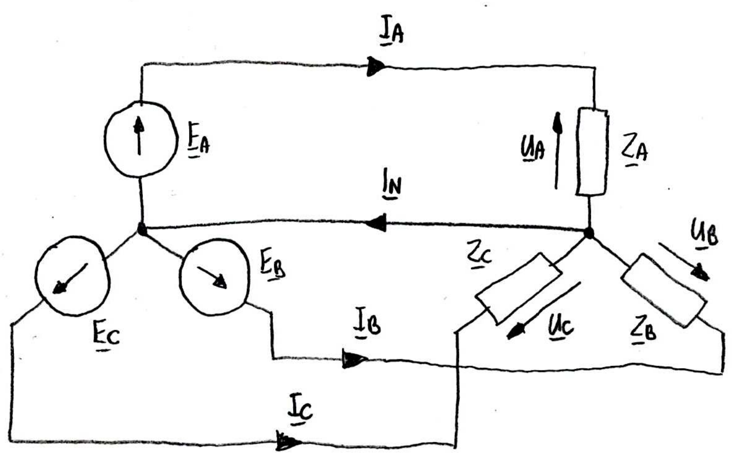


$$\underline{E}_A = 230\,e^{j0^\circ} = 230\ \mathrm{V}$$

$$\underline{E}_B = 230\,e^{-j120^\circ} = (-115 - j199.19)\ \mathrm{V}$$

$$\underline{E}_C = 230\,e^{+j120^\circ} = (-115 + j199.19)\ \mathrm{V}$$

$$\underline{U}_A = \underline{E}_A \qquad \underline{U}_B = \underline{E}_B \qquad \underline{U}_C = \underline{E}_C$$

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_A} = \frac{230}{6+j8} = (13.8 - j18.4) = 23\,e^{-j53.13^\circ}\ \mathrm{A}$$

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_B} = \frac{(-115 - j199.19)}{8+j6} = (-21.15 - j9.04)\ \mathrm{A} = 23\,e^{-j156.87^\circ}\ \mathrm{A}$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \frac{(-115 + j199.19)}{20} = (-5.75 + j9.96)\ \mathrm{A} = 11.5\,e^{+j120^\circ}\ \mathrm{A}$$

The rated current of the overcurrent circuit breaker:

$$\boxed{I_n = 25\ \mathrm{A}}$$

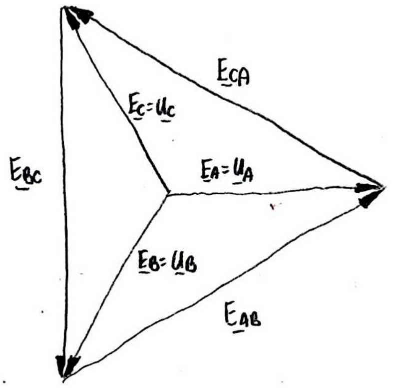
$\underline{I_N} = \underline{I_A} + \underline{I_B} + \underline{I_C} = (13.8 - j\,18.4) + (-21.15 - j\,9.04) + (-5.75 + j\,9.86) = (-13.10 - j\,17.48)\,A = 21.84\,e^{-j126.86°}\,A$

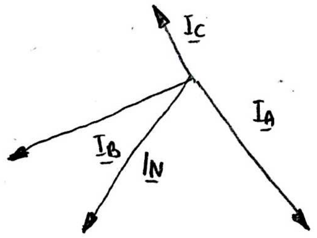
$P_{TOTAL} = Re\{\underline{Z_A}\} \cdot I_A^2 + Re\{\underline{Z_B}\} \cdot I_B^2 + Re\{\underline{Z_C}\} \cdot I_C^2 = 6 \cdot 23^2 + 8 \cdot 23^2 + 20 \cdot 11.5^2 = 10051\ W$

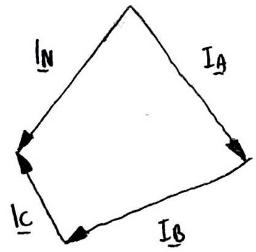
$W = P_{TOTAL} \cdot time = 10051 \cdot 8.5 = 402.040\ Wh = 402.04\ kWh$

$cost = 402.04 \cdot 0.289 = \boxed{116.19\ Euro}$

$\underline{I_A} + \underline{I_B} + \underline{I_C} = \underline{I_N}$

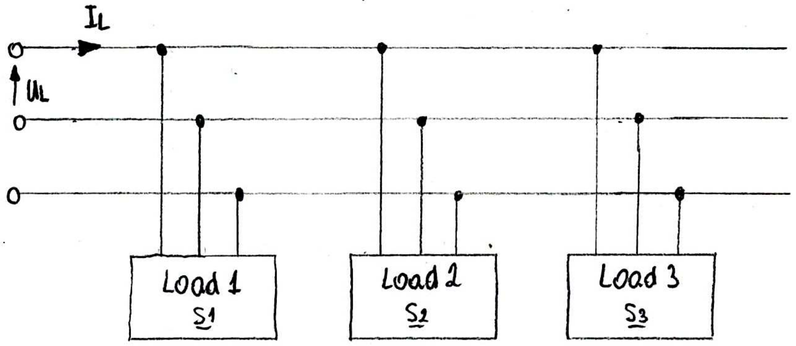






PROBLEM #2

A balanced 3-phase distribution line is used to supply three balanced Y-loads that are connected in parallel: Loaad 1: 37 kVA at 0.72 pf, lagging
Load 2: 64 kVA at 0.83 pf, leading
Load 3: 55 kW and 28 kVAr.
The line voltage at the load is 660V rms. Find the line current in the distribution line and the combined power factor (pf) at the load.



$\underline{S} = P + jQ \quad P = S \cdot \cos\varphi \quad Q = S \cdot \sin\varphi \quad S = \sqrt{P^2 + Q^2} \rightarrow Q = \sqrt{S^2 - P^2}$

<u>Load 1</u>

$S_1 = 37000\ VA \quad \cos\varphi_1 = 0.72$

$P_1 = S_1 \cdot \cos\varphi_1 = 37000 \cdot 0.72 = 26640\ W \qquad Q_1 = \sqrt{S_1^2 - P_1^2} = \sqrt{37000^2 - 26640^2} = 25677\ VAr$

$\underline{S_1} = P_1 + jQ_1 = (26640 + j\,25677)\ VA$ pf is lagging $\rightarrow Q > 0$

<u>Load 2</u>

$S_2 = 64000\ VA \quad \cos\varphi_2 = 0.83$

$P_2 = S_2 \cdot \cos\varphi_2 = 64000 \cdot 0.83 = 53120\ W \qquad Q_2 = \sqrt{S_2^2 - P_2^2} = \sqrt{64000^2 - 53120^2} = 35697\ VAr$

$\underline{S_2} = P_2 - jQ_2 = (53120 - j\,35697)\ VA$ pf is leading $\rightarrow Q < 0$

Load 3

$P_3 = 55\,000\, W \quad Q_3 = 28000\ VAr$

$\underline{S}_3 = P_3 + jQ_3 = (55000 + j\,28000)\ VA$

S_T - total apparent power

$\underline{S}_T = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 = 26\,640 + j\,25677 + 53120 - j\,35687 + 55000 + j\,28000 = (134\,760 + j\,18880)\ VA =$

$= 136\,080\, e^{j\,8.02^\circ}\ VA$

In the case of balanced load

$S_T = 3 \cdot V_p \cdot I_p \quad V_p$ - phase voltage, I_p - phase current

or

$\underbrace{S_T = \sqrt{3} \cdot V_L \cdot I_L}$ $\quad V_L$ - line voltage, $\quad I_L$ - line current

$\Downarrow$

$I_L = \dfrac{S_T}{\sqrt{3} \cdot V_L} = \dfrac{136\,080}{\sqrt{3} \cdot 660} = \boxed{119.05\ A}$

Combined power factor

$\varphi = 8.02^\circ \quad \cos\varphi = \cos 8.02^\circ = 0.99$

$pf_{load} = \boxed{0.99}$

PROBLEM #3

A 3-phase electric heater with nominal power $P_n = 15\,kW$ is used to heat the warehouse. The heater is supplied from a power network with a phase voltage of $U_n = 230\,V$. The heating elements are Δ-connected. The heater operates at full power for 12 hours a day. A photovoltaic installation has been installed in close proximity to the warehouse, which has caused the phase voltage increase from $U_n = 230\,V$ to $U_n' = 242\,V$. Calculate the percentage increase in warehouse heating costs in one day due to the increase in phase voltage.

The energy consumption: $W = P_n \cdot t = 15\,000 \cdot 12 = 180\ kWh$

Power of one phase of the heater: $P_1 = \dfrac{P_n}{3} = \dfrac{15000}{3} = 5\ kW$

$P_1 = U_{phase} \cdot I_{phase} \cdot \cos\varphi = \dfrac{U_{phase}^2}{R} \cdot \cos\varphi$

$U_{phase} = U_L = \sqrt{3}\, U_n$

$\cos\varphi = 1$ (the heater is resistive in nature)

$R = \dfrac{U_{phase}^2}{P_1} = \dfrac{(\sqrt{3}\, U_n)^2}{P_1} = \dfrac{(\sqrt{3} \cdot 230)^2}{5000} = 31.74\ \Omega$

$U_n = 230\,V \rightarrow U_n' = 242\,V$ (increase by 5.2%)

Power of one phase of the heater: $P_1' = \dfrac{U_L'^2}{R} = \dfrac{(\sqrt{3} \cdot U_n')^2}{R} = \dfrac{(\sqrt{3} \cdot 242)^2}{31.74} = 5535.35\ W$

$W' = 3 \cdot 12 \cdot P_1' = 3 \cdot 12 \cdot 5535.35 = 199.27\ kWh$

The percentage increase in costs:

$\dfrac{W' - W}{W} \cdot 100\% = \dfrac{199.27 - 180}{180} \cdot 100\% = \boxed{10.71\%}$

PROBLEM #4

In the room, there is a three-phase electric motor supplied from a power network with a voltage of $U_n = 230/400$ V. The motor windings are Δ-connected. The motors parameters are: nominal power $P_1 = 15$ kW, motor efficiency $\eta_1 = 0.885$, power factor $\cos\varphi_1 = 0.9$. It was decided to install a second three-phase motor in the same room with the parameters: $P_2 = 7.5$ kW, $\eta_2 = 0.88$, $\cos\varphi_2 = 0.88$. The room is supplied with power by a copper wire $5\times6\,mm^2$. Check whether, after installing the second motor, the cable cross-section will be sufficient due to its long-term current carrying capacity. In the case of 5-core copper conductors laid in the room, the long-term current carrying capacities are: $1.5\,mm^2$ - 17 A, $2.5\,mm^2$ - 24 A, $4\,mm^2$ - 31 A, $6\,mm^2$ - 40 A, $10\,mm^2$ - 55 A.

$$\eta = \frac{\text{mechanical power}}{\text{electrical power}} \Rightarrow \text{electrical power} = \frac{\text{mechanical power}}{\eta}$$

$$P_{1e} = \frac{P_1}{\eta_1} = \frac{15\,000}{0.885} = 16759.8\ W$$

$$P_{2e} = \frac{P_2}{\eta_2} = \frac{7500}{0.88} = 8522.7\ W$$

$$S = \frac{P}{\cos\varphi} \qquad Q = \sqrt{S^2 - P^2}$$

$$S_1 = \frac{P_{1e}}{\cos\varphi_1} = \frac{16759.8}{0.9}\ 18\,622\ VA \qquad Q_1 = \sqrt{S_1^2 - P_{1e}^2} = \sqrt{18622^2 - 16759.8^2} = 8117.1\ VAr$$

$$S_2 = \frac{P_{2e}}{\cos\varphi_2} = \frac{8522.7}{0.88} = 9684.9\ VA \qquad Q_2 = \sqrt{S_2^2 - P_{2e}^2} = \sqrt{9684.9^2 - 8522.7^2} = 4600\ VAr$$

$$\underline{S}_1 = P_{1e} + jQ_1 = (16759.8 + j\,8117.1)\ VA$$

$$\underline{S}_2 = P_{2e} + jQ_2 = (8522.7 + j\,4600)\ VA$$

$$\underline{S}_T = \underline{S}_1 + \underline{S}_2 = (25282.5 + j\,12717.1)\ VA \qquad S_T = |\underline{S}_T| = 28\,300\ VA$$

$$S = 3 \cdot U_{phase} \cdot I_{phase} = \sqrt{3} \cdot U_{line} \cdot I_{line}$$

$$I_{line} = \frac{S}{\sqrt{3}\,U_{line}} = \frac{28300}{\sqrt{3}\cdot\sqrt{3}\cdot 230} = \boxed{41.02\ A}$$

The cross section of the cable supplying the motors will be too small.