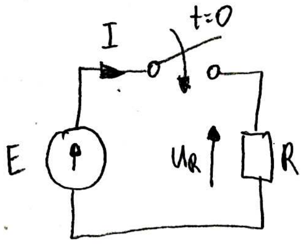
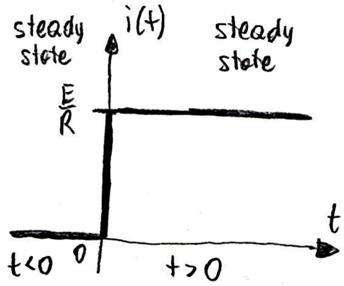
ELECTRICAL CIRCUITS 2 - CLASS NO. 10 (21.05.2024)

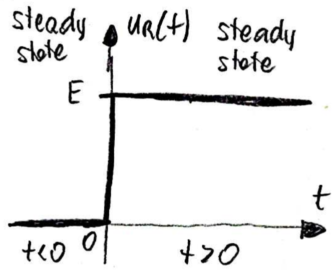
TRANSIENT AND STEADY STATE

* R CIRCUIT



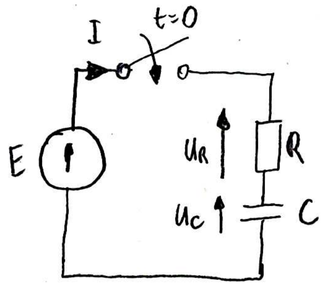
$t<0$	$t>>0$
$I=0$	$I=\frac{E}{R}$
$U_R=0$	$U_R=E$



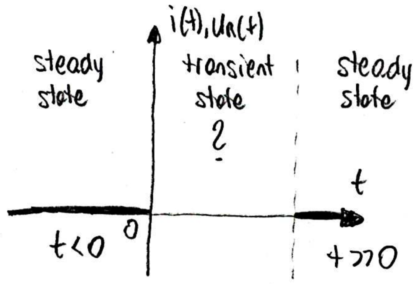


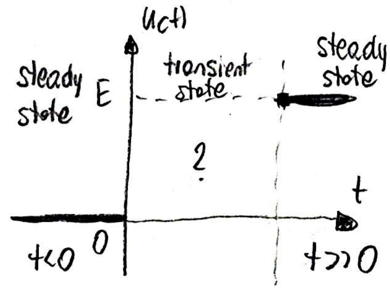
E - DC voltage source

* RC CIRCUIT

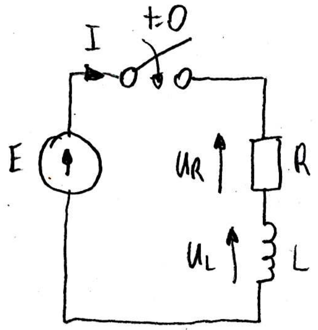


$t<0$	$t>>0$
$I=0$	$I=0$
$U_R=0$	$U_R=0$
$U_C=0$	$U_C=E$

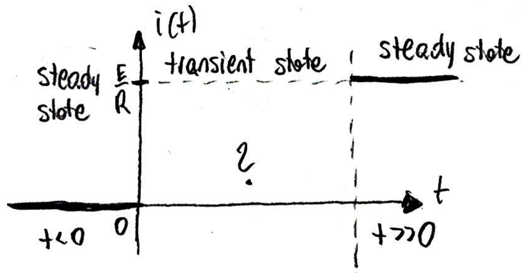


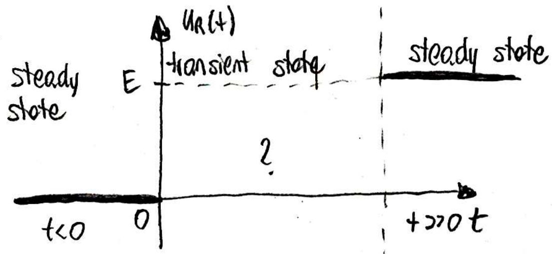


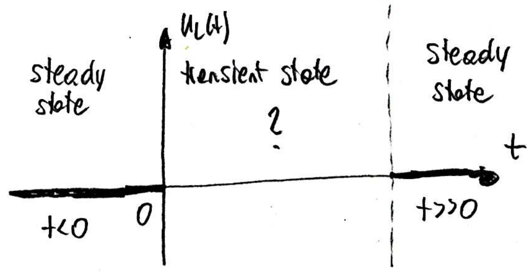
* RL CIRCUIT



$t<0$	$t>>0$
$I=0$	$I=\frac{E}{R}$
$U_R=0$	$U_R=E$
$U_L=0$	$U_L=0$



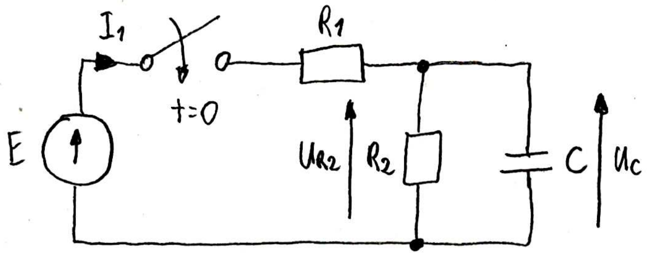




INITIAL AND STEADY STATE VALUSES

* initial conditions
  - for C → U_C at $t=0$
  - for L → I_L at $t=0$

* example 1



$t<0$

$\boxed{U_C = 0}$

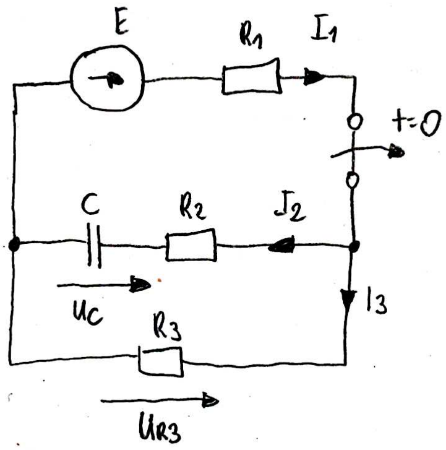
$t>>0$

$U_C = U_{R2}$

$I_1 = \frac{E}{R_1+R_2}$ $\quad U_{R2} = U_C = I_1 \cdot R_2 = \frac{R_2 \cdot E}{R_1+R_2}$

$\boxed{U_C = \frac{R_2 \cdot E}{R_1+R_2}}$

* example 2



$t<0$

$U_C = U_{R3}$

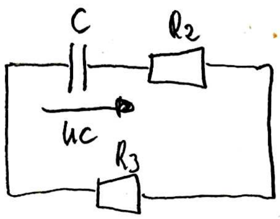
$I_2 = 0$, ↪ open circuit for DC

$I_1 = I_3$

$I_1 = \frac{E}{R_1+R_3}$ $\quad U_{R3} = I_1 R_3 = \frac{R_3 E}{R_1+R_3}$

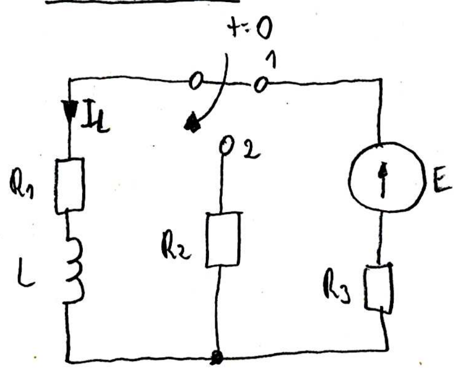
$\boxed{U_C = \frac{R_3 \cdot E}{R_1+R_3}}$

$t>>0$



no source → $\boxed{U_C = 0}$

* example 3

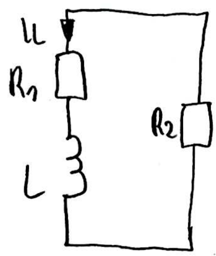


$t<0$

L → short circuit for DC

$\boxed{I_L = \frac{E}{R_1+R_3}}$

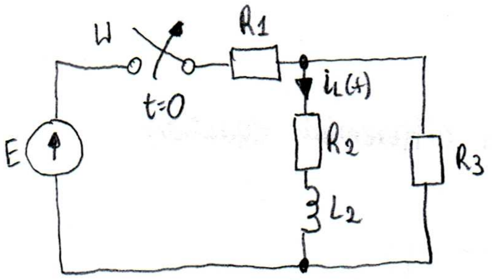
$t>>0$



no source → $\boxed{I_L = 0}$

PROBLEM #1

The circuit shown in the figure has been in a steady-state. The switch was open at $t=0$. Find $i_L(t)$. $E=80V$, $R_1=120\Omega$, $R_2=50\Omega$, $R_3=200\Omega$, $L_2=0.75\,H$.

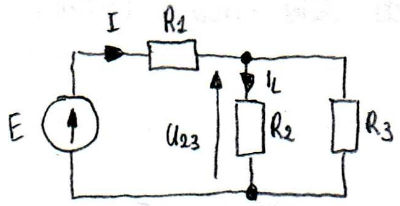


* INITIAL VALUES ($t<0$), steady-state before opening the switch

Notes:

- we consider circuit as a DC circuit
- in the case of a DC circuit, we replace inductor with a short-circuit and capacitor with an open-circuit
- we use classical methods: Ohm's Law, KCL, KVL

our circuit before opening the switch (DC circuit), $I_L=?$



$$I=\frac{E}{R_1+\frac{R_2\cdot R_3}{R_2+R_3}}=\frac{80}{120+\frac{50\cdot 200}{50+200}}=\frac{80}{120+40}=\frac{80}{160}=0{,}5\,A$$

$$U_{23}=I\cdot\frac{R_2\cdot R_3}{R_2+R_3}=0.5\cdot\frac{50\cdot 200}{50+200}=0.5\cdot 40=20\,V$$

$$I_L=\frac{U_{23}}{R_2}=\frac{20}{50}=0.4\,A$$

* TRANSIENT ANALYSIS ($t>0$), circuit after opening the switch

Notes:

- we write differential equation with the use of KVL or KCL
- note that: $u_L=L\frac{di_L(t)}{dt}$ $\quad i_C(t)=C\frac{du_C(t)}{dt}$
- general form of differential equation:

$$a_n\frac{d^n x(t)}{dt^n}+a_{n-1}\frac{d^{n-1}x(t)}{dt^{n-1}}+\ldots+a_1\frac{dx(t)}{dt}+a_0x(t)=f(t) \qquad (1)$$

where:

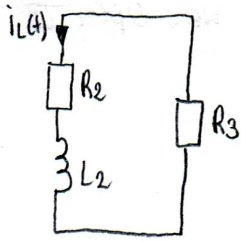
$x(t)$ - searched quantity (current / voltage)

$a_n, a_{n-1},\ldots, a_1, a_0$ - constant coefficients (R, L, C, M in the circuit)

$f(t)$ - voltage and current sources

n - the order of differential equation (the number of inductors / capacitors in the circuit)

Our circuit after opening the switch



using KVL:

$$u_{R2}(t) + u_{L2}(t) + u_{R3}(t) = 0$$

$$R_2 \cdot i_L(t) + L_2 \frac{di_L(t)}{dt} + R_3 \cdot i_L(t) = 0$$

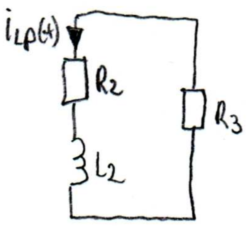
$$\boxed{L_2 \frac{di_L(t)}{dt} + (R_2+R_3)\, i_L(t) = 0}$$ - a first-order differential equation

> Notes:
>
> - the solution of equation (1) has a form
>
> $$x(t) = x_p(t) + x_c(t) \qquad (2)$$
>
> where:
>
> $x_p(t)$ - the particular integral solution (forced response)
>
> $x_c(t)$ - the complementary solution (natural response)

in our case: $i_L(t) = i_{Lp}(t) + i_{Lc}(t)$

> Notes:
>
> - the particular integral solution we obtain by solving the steady state circuit ($t>>0$)

in steady state ($t>>0$) $i_{Lp}(t) = ?$



$i_{Lp}(t) = 0$ ← because there is no voltage source in the circuit

> Notes:
>
> - we obtain the complementary solution by solving the homogeneous equation (1)
>
> $$a_n \frac{d^n x_c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x_c(t)}{dt^{n-1}} + \ldots + a_1 \frac{dx_c(t)}{dt} + a_0 x_c(t) = 0 \qquad (3)$$
>
> - the solution of (3) has a form:
>
> $$x_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \ldots A_n e^{s_n t}$$
>
> where:
>
> $s_1, s_2, \ldots, s_n$ - roots of characteristic equation: $a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 = 0$
>
> $A_1, A_2, \ldots A_n$ - integration constants determined with the use of initial values

and

$i_{Lc}(t)$ - the complementary solution

a first-order differential equation:

$$\boxed{L_2 \frac{d i_{Lc}(t)}{dt} + (R_2+R_3)\, i_{Lc}(t) = 0}$$

has a solution in the form:

$$\boxed{i_{Lc}(t) = A_1 \cdot e^{S_1 t}}$$

S_1 is determined from the characteristic equation:

$$L_2 \cdot S_1 + (R_2+R_3) = 0 \Rightarrow \boxed{S_1 = -\frac{R_2+R_3}{L_1} = -\frac{50+200}{0.75} = -333.33}$$

A_1 is determined with the use of initial conditions

at $t=0$: $i_L(0^-) = i_L(0^+)$ $\qquad i_L(0^-) = i_L(0^+) = I_L = 0.4\,A$

$$i_L(t) = i_{Lp}(t) + i_{Lc}(t)$$

$$i_L(t) = 0 + A_1 e^{S_1 t}$$

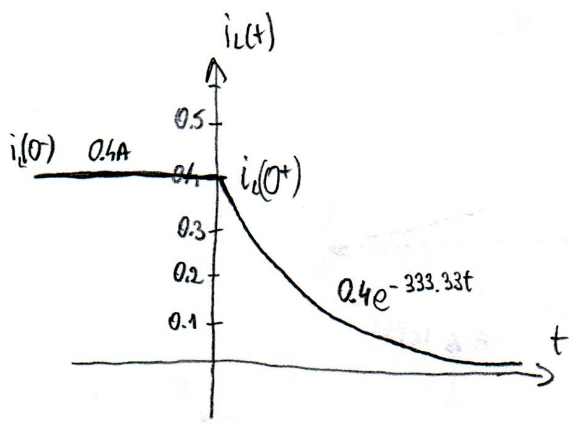
$$i_L(t) = A_1 e^{-\frac{R_2+R_3}{L_2} \cdot t}$$

$$t=0 \Rightarrow 0.4 = A_1 \underbrace{e^{-\frac{R_2+R_3}{L_2} \cdot 0}}_{=1}$$

$$\boxed{A_1 = 0.4}$$

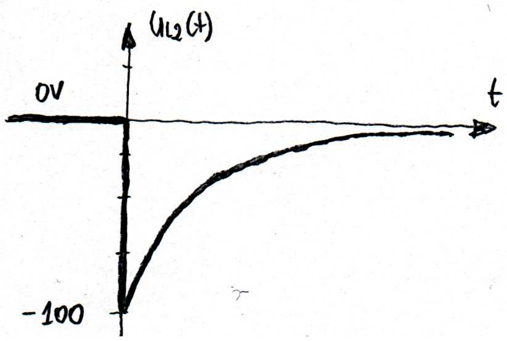
finally:

$$i_L(t) = i_{Lp}(t) + i_{Lc}(t) = 0 + 0.4 e^{-333.3t} = \boxed{0.4 e^{-333.3t}\ A}$$



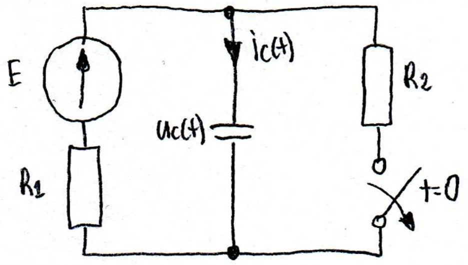
$$u_{L2}(t) = L_2 \frac{d i_L(t)}{dt} = 0.75 \frac{d}{dt}\left(0.4 e^{-333.3t}\right) =$$

$$= 0.75 \cdot 0.4 \cdot (-333.3) e^{-333.3t} = \boxed{-100 e^{-333.3t}}$$

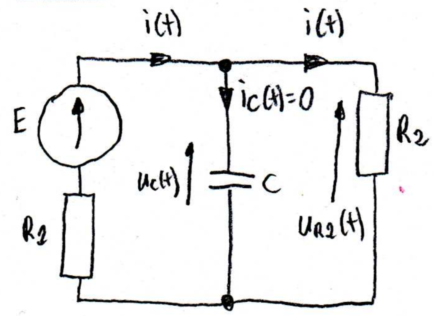


PROBLEM #2

The circuit shown in the figure has been in a steady-state. The switch was open at $t=0$. Calculate and plot $i_C(t)$ and $u_C(t)$ for $t<0$, $t=0$ and $t>0$. $E=100V$, $R_1=20\Omega$, $R_2=40\Omega$, $C=10mF$.



steady-state before opening the switch ($t<0$)



$i_C(t)=0A$

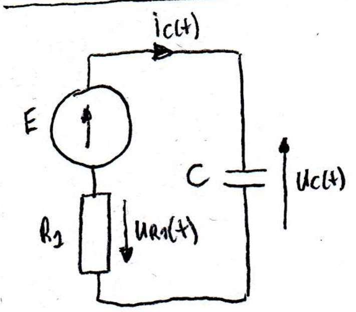
$u_C(t)=u_{R2}(t)$

$i(t)=\frac{E}{R_1+R_2}$

$u_{R2}(t)=R_2\cdot i(t)=\frac{R_2}{R_1+R_2}\cdot E$

$u_{R2}(t)=\frac{40}{20+40}\cdot 100=66{,}67\ V$

circuit after opening the switch ($t>0$)



$u_{R1}(t)+u_C(t)=E$

$R_1\cdot i_C(t)+u_C(t)=E$

$i_C(t)=C\frac{du_C(t)}{dt}$

$$\boxed{R_1C\frac{du_C(t)}{dt}+u_C(t)=E}$$

$$\boxed{u_C(t)=u_{Cp}(t)+u_{Cc}(t)}$$

$u_{Cp}(t)$ - the particular solution ($t\gg0$)

$$\boxed{u_{Cp}(t)=E=100\ V}$$

$u_{Cc}(t)$ - the complementary solution

$R_1C\frac{du_{Cc}(t)}{dt}+u_{Cc}(t)=0$ $\qquad u_{Cc}(t)=A_1e^{s_1t}$

$R_1Cs_1+1=0 \Rightarrow s_1=-\frac{1}{R_1C}=-\frac{1}{20\cdot 0.01}=-\frac{1}{0.2}=-5\ s$

at $t=0$

$u_C(t)=u_{Cp}(t)+u_{Cc}(t)$

$66{,}67=100+A_1e^0 \Rightarrow A_1=66{,}67-100=-33{,}33\ V$

$$\boxed{u_{Cc}(t)=-33.33e^{-5t}\ V}$$

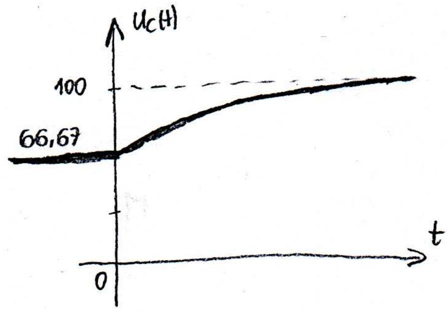
$u_C(t)=u_{Cp}(t)+u_{Cc}(t)=\boxed{100-33{,}33e^{-5t}\ V}$

$i_C(t)=C\frac{du_C(t)}{dt}=0.01\frac{d}{dt}\left(100-33{,}33e^{-5t}\right)=0.01\cdot 33{,}33\cdot 5e^{-5t}=\boxed{1.67e^{-5t}\ A}$

$u_C(t<0)=66.67\ V$

$u_C(t=0)=66.67\ V$

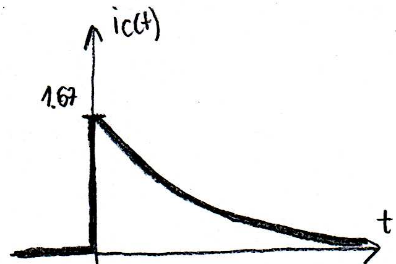
$u_C(t\to\infty)=100\ V$



$i_C(t<0)=0\ A$

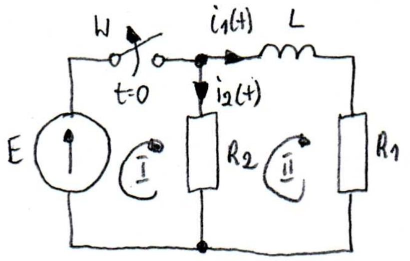
$i_C(t=0)=1.67\ A$

$i_C(t\to\infty)=0A$



PROBLEM #3

The circuit shown in the figure has been in a steady-state. The switch was open at $t=0$. Plot $i_1(t)$ and $i_2(t)$ for $t<0$, $t=0$ and $t>0$. Use the classical method. $E=100V$, $L=0.1H$, $R_1=25\,\Omega$, $R_2=75\,\Omega$.



steady-state before opening switch $(t<0)$

$$\begin{cases} I_I R_2 - I_{II} R_2 = E \\ -I_I R_2 + I_{II}(R_1+R_2) = 0 \end{cases}$$

$$\begin{cases} 75 I_I - 75 I_{II} = 100 \\ -75 I_I + 100 I_{II} = 0 \quad + \end{cases}$$

$$25 I_{II} = 100$$

$$I_{II} = 4\,A$$

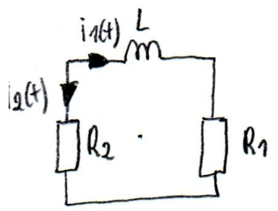
$$I_I R_2 = E + I_{II} R_2$$

$$I_I = \frac{E + I_{II} R_2}{R_2} = \frac{100 + 4\cdot 75}{75} = \frac{400}{75} = 5.33\,A$$

$$i_1 = I_{II} = \boxed{4A}$$

$$i_2 = I_I - I_{II} = 5{,}33 - 4 = \boxed{1{,}33\,A}$$

circuit after opening the switch $(t>0)$



using KVL:

$$i_1(t)\cdot R_1 + i_1(t)\cdot R_2 + L\frac{di_1(t)}{dt} = 0$$

$$L\frac{di_1(t)}{dt} + (R_1+R_2)\,i_1(t) = 0$$

$$i_1(t) = i_{1p}(t) + i_{1c}(t)$$

$i_{1p}(t)$ – the particular solution

$$i_{1p}(t) = 0$$

$i_{1c}(t)$ – the complementary solution

$$L\frac{di_{1c}(t)}{dt} + (R_1+R_2)\,i_{1c}(t) = 0 \qquad Ls_1 + R_1 + R_2 = 0 \qquad s_1 = -\frac{R_1+R_2}{L}$$

$$i_{1c} = A_1 e^{s_1 t}$$

at $t=0$: $\quad i_1(t) = i_{1p}(t) + i_{1c}(t)$

$$4 = 0 + A_1 e^0 \Rightarrow A_1 = 4$$

$$i_1(t) = 4e^{-1000t}\,A$$

$$i_2(t) = -i_1(t) = -4e^{-1000t}\,A$$